Higher order strong-coupling corrections combined with the weak-coupling-plus model and a realistic quasiparticle scattering amplitude can account for the magnitudes of the specific heat jumps of superfluid $^3$He at high pressure. These higher order effects do not destroy the agreement between the weak-coupling-plus model and the B phase specific heat measurements of Alvesalo, et al.

Rainer and Serene$^1$ (hereafter RS) showed that to lowest order in $T_c/T_F$ the strong-coupling corrections to the free energy of superfluid $^3$He can be expressed in terms of weighted angular averages of the normal state scattering amplitude for quasiparticles interacting on the Fermi surface. This same scattering amplitude appears in Landau's theory of the normal Fermi liquid and determines the low temperature transport coefficients.

Recently scattering amplitudes which account for the measured Landau parameters and transport coefficients have been proposed.$^{2,3,4}$ However, quantitative agreement between measured and calculated specific heat jumps, using these scattering amplitudes, can only be obtained if higher order strong-coupling corrections are included in the calculations. These corrections come from high energy cutoffs in the frequency sums appearing in the RS theory, and are due to the quasiparticle lifetime and frequency dependences of the pairing interaction and scattering amplitude. There are also corrections from diagrams in the free energy expansion that are explicitly higher order in $T_c/T_F$ (eg. Fig. 6 of Ref. 1). Unlike the leading order strong-coupling corrections calculated by RS, the higher order effects cannot be systematically calculated in the framework of Landau's Fermi liquid theory. Nevertheless, higher order strong-coupling corrections are important in $^3$He and when accounted for yield reasonable agreement between the theory and measured specific heat jumps.

High energy cutoffs in the frequency sums of the RS theory produce large effects on the calculated specific heat jumps. To illustrate this we note that the strong-coupling corrections to the Ginzburg-Landau parameters calculated by RS have the form

$$
\Delta S_1 = \Delta S_1^f + \Delta S_1^{b+c} + \Delta S_1^d,
$$

with

$$
\Delta S_1^f = -\hat{\eta}_1 S_a \Delta \lambda^f_1 \quad (a = f, b+c, d),
$$

where $\hat{\eta}_n$ are constants, $\Delta \lambda^f_1$ are angular averages of the quasiparticle scattering rate (see Eqs. 3.31 of Ref. 1), and $S_a$ are frequency sums over superfluid corrections to the quasiparticle propagators:

$$
S_{b+c} = (2\pi k_B T)^4 \sum_{n_3} \sum_{n_2} \sum_{n_1} \sum_{n_2} \sum_{n_3} \sum_{n_4} \delta_{n_3} \delta_{n_2} \delta_{n_1} \delta_{n_2} \delta_{n_3} \delta_{n_4}.
$$

In $^3$He these sums are cutoff at high frequency by the frequency dependence of the pairing interaction, etc. The summations extend over the integers, and $\delta_{n_4} = n_1 + n_2 - n_3$. To leading order in $T_c/T_F$ $\delta_{n_4}$ can be replaced by $\delta/(2\pi k_B T) = 1.1$. For this case the frequency sums can be calculated analytically:

$$
S_1^{b+c} = \pi^2 \text{ln}(2) = 6.84, \quad S_1^d = 5\pi^4/48 = 10.15, \quad \text{and} \quad S_1^f = 5\pi^4/16 = 30.44.
$$

However, these frequency sums are slowly converging.

In $^3$He these sums are cutoff at high frequency by the frequency dependence of the pairing interaction, etc. Since the sums are otherwise slowly converging the frequency cutoff can be important even if the effective cutoff energy $\epsilon_C \gg k_B T_c$. Estimates of $\epsilon_C$ based on $T_c$ suggest that $\epsilon_C \lesssim 0.1 k_B T_c$. To further emphasize the weak convergence of the frequency sums and to demonstrate the importance of a finite high-frequency cutoff, we show in Fig. 1 the frequency sums calculated with a smooth cutoff $\Delta(\epsilon_n) = 1/(1+(\epsilon_n/\epsilon_C)^4)$. The cutoff effects both the magnitude and the relative weights of the b, c, and f contributions to the Ginzburg-Landau parameters. The largest effect is on the feedback corrections $\Delta S_{b+c}$, which are the most important contributions determining the relative stability of the axial and isotropic states. Compared to measurements of the A- and B-phase specific heat jumps, the axial state is over-stabilized relative to the isotropic state if we neglect the high frequency cutoff. This is illustrated in Fig. 2 where we show the specific heat jumps, calculated for melting pressure, as a function of the frequency cutoff. The scattering amplitude used in the calculation accurately fits the known Landau parameters and normal state transport coefficients and is discussed in detail in Ref. 2. For sufficiently small values...
of the cutoff the feedback corrections no longer stabilize the axial state. If we choose \( x_\epsilon = 0.25 \) (\( \epsilon_\epsilon = 0.068 \) kBT_c), the difference \( \Delta \beta_{12} + 1/3 \Delta \beta_{345} = \Delta \beta_{245} \) (the most sensitive measure of the axial state stability) gives the experimental value of 0.39, and the specific heat jumps are 8% (12%) larger than those of Ref.7 (Ref.6).

Strong coupling corrections also modify the temperature dependence of the specific heat. The weak-coupling-plus model agrees well with the B-phase specific heat measurements of Alvesalo, et al. Since the frequency cutoff affects the calculated specific heat discontinuities significantly, we have calculated the temperature dependent specific heat for the isotropic state using the same frequency cutoff. Our calculation shown in Fig.3 is the same as the weak-coupling-plus calculation except that \( A_0(T) \) is replaced by \( A_0(T)(\epsilon_\epsilon) \) in Eqs.(A1) through (A5) of Ref.8. The frequency cutoff has a minor effect provided the scattering rates in the two calculations give the same specific heat jump. The result is the same if the diagram in Fig. 6 of RS is included in the free energy functional. In addition the frequency cutoff significantly reduces the contribution from this diagram relative to the contributions from the weak-coupling-plus diagrams.

Finally, we emphasize that \( \epsilon_\epsilon \) is a phenomenological parameter, which we have used to cutoff the frequency sums in Eqs.(1). The actual frequency dependence of the pairing interaction, scattering amplitude, and propagators are known imprecisely. Consequently, calculations based on an assumed pressure dependence of \( \epsilon_\epsilon \) that yield the observed pressure for the PCP, although interesting, may be fortuitous.

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REFERENCES