P-WAVE SUPERFLUIDITY IN NEUTRON STARS AND $^3$He

J.A. Sauls and D.L. Stein

Joseph Henry Laboratories of Physics
Princeton University, Princeton, New Jersey 08544

We discuss some of the similarities and differences between p-wave pairing in $^3$He and the $^3p$ neutron superfluid that is believed to exist in the interiors of neutron stars. The structure of vortices in the $^3p_2$ neutron superfluid is particularly interesting and is relevant to the astrophysics of pulsars.

Like superfluid $^3$He, the neutron fluid in the interior of neutron stars is believed to be a spin-triplet, p-wave superfluid. In contrast to $^3$He, there is a strong spin-orbit interaction in neutron matter. At densities $\rho > 1.5 \times 10^{14} g/cm^3$ this interaction favors condensation of neutron pairs in a $^3P_2$ state ($S=1, L=1$, and total angular momentum $J=2$).

Both superfluid $^3$He and the neutron superfluid are described by a complex $3 \times 3$ matrix order parameter $A_{\mu\nu}$. For the $^3P_2$ case $A_{\mu\nu}$ is traceless and symmetric because of the $J=2$ spin-orbit coupling. The equilibrium phase of the $^3P_2$ superfluid is found by minimizing the Ginzburg-Landau (GL) free energy functional. This problem has been solved by recognizing the formal equivalence between the GL theory for $^3P_2$ pairing and the $d$-wave pairing problem solved by Mermin. The fourth order GL functional leads to three types of phases, one of which (the type I phase) has flow properties similar to those of $^3$He-A. The order parameter is

$$A_{\mu\nu} = e^{iX}[u_{\mu} u_{\nu} - \frac{1}{3} \delta_{\mu\nu}],$$

where $\lambda$ is the amplitude and $u \cdot \dot{v} = 0$. A type I phase in rotating neutron stars would have interesting rotational dynamics. For example, the topology of this phase allows for $8\pi$ coreless vortices similar to the Anderson-Toulouse $4\pi$ coreless vortices of $^3$He-A.

Within the BCS theory the type I phase is not the absolute free energy minimum. The example of $^3$He suggests that strong-coupling effects must be considered, particularly since $T_c/T_F$, which sets the scale for the important strong-coupling corrections, is expected to be of the same order of magnitude in neutron matter as in $^3$He. Strong-coupling corrections, based on the s-p approximation for neutron matter, do not modify the BCS prediction that the free energy is minimized by any real (up to an overall phase), traceless, symmetric $A_{\mu\nu}$. In particular

$$A_{\mu\nu} = A_{\mu\nu} = \lambda e^{iX}[u_{\mu} u_{\nu} + r v_{\mu} v_{\nu} - (1+\gamma)w_{\mu} w_{\nu}],$$

where $-1 \leq r \leq -\frac{1}{3}$ parameterizes the excess degeneracy of the real $^3P_2$ states and $(u,v,w)$ are orthonormal eigenvectors. This degeneracy is broken by the sixth-order terms in the GL expansion which favor the $r=-\frac{1}{3}$ state with an axial symmetric order parameter

$$A_{\mu\nu} = e^{iX}[u_{\mu} u_{\nu} - \frac{1}{3} \delta_{\mu\nu}],$$

that describes $^3P_2$ Cooper pairs in a pure $m_J=0$ state with $\pi$ as the quantization axis.

Because of the near degeneracy of the states described by eq. (2), the equilibrium state of the $^3P_2$ superfluid is sensitive to small perturbations. In a magnetic field the Cooper pairs orient so that the eigenvector $\omega$ with smallest eigenvalue aligns along the field direction. In addition, for fields $H \gtrsim 10^{15}$ gauss, which are possible in neutron stars, the magnetic free energy $\Delta Q_H = g_\mu B (\omega)\mu_H$ dominates the sixth-order free energy and stabilizes the $r = -1$ state with

$$A_{\mu\nu} = e^{iX}[u_{\mu} u_{\nu} - v_{\mu} v_{\nu}],$$

in which the pairs condense with equal amplitudes in states with $m_J = \pm 2$ along $\omega$.

Non-uniform states are particularly interesting in the $^3P_2$ superfluid. A superfluid described by eq. (4) has topologically stable vortices with half the circulation of a conventional Fermi superfluid. Furthermore a single half-quantum vortex, which corresponds to a phase change by $\pi$ and a rotation of $\dot{u}$ and $\dot{v}$ by $\pi/2$, has only half the kinetic and bending energies of a conventional $2\pi$ phase vortex. Similar half-quantum vortices are allowed by the planar state which is proposed to exist very close to the superfluid transition temperature in $^3$He. In a weak magnetic field, the existence of a continuous family of nearly degenerate states interpolating between (3) and (4) has interesting implications for non-uniform states; in particular, it leads to an unusual vortex configuration.

For a vortex line in He-II, the phase changes by $2\pi$ around a vortex line. The only structure allowed in such a vortex is a vanishing amplitude for distances $r < \xi_T$, the temperature-dependent coherence length. This is the only mechanism available for reducing the kinetic energy associated with the divergent velocity field. However, the $^3P_2$ vortex structure is complicated by both the existence of other relative minima of the GL functional and by the excess degeneracy of the real $^3P_2$ states. There is an additional length scale $\xi_r \gg \xi_T$ at which the kinetic energy density is comparable to the
energy difference between the real $^3P_2$ states. For $\rho \gg \xi_T$, the $^3P_2$ vortex is given by (3)
with $\hat{u}$ parallel to the vortex line and $\gamma$ equal to the azimuthal angle in cylindrical coordinates. At intermediate distances $\xi_T < \rho < \xi_T$, the gradient energy dominates the sixth-order free energy but is still small compared to the total condensation energy. The gradient energy resolves the excess degeneracy by selecting the state which minimizes the sum of the kinetic and bending energies. In this case the vortex is described by

$$A_{\mu \nu}(\hat{R}) = e^{i\hat{\Phi}}[\hat{u} \hat{\xi} + \hat{\rho} \hat{\rho} - (1+\rho)\hat{\xi}\hat{\xi}]$$

with $r = 6 - \sqrt{3} = 0.5574$ and $(\rho, \phi, z)$ are cylindrical coordinates measured from the vortex line. This state describes a $2\pi$ disgyration and a $2\pi$ phase vortex. Near the vortex core ($\rho < \xi_T$) the gradient energy is comparable to the condensation energy, so the order parameter may deviate from eq. (2) to lower the total energy. In the region $\rho < \xi_T$ the order parameter is a linear combination of a real $^3P_2$ state and a type I state. The general form of the vortex order parameter is

$$A_{\mu \nu}(\hat{R}) = \frac{\Delta}{\sqrt{2}} e^{i\hat{\Phi}}(f_1(\rho)\hat{\rho} + f_2(\rho)\hat{\xi})$$

where $f_1(\rho)$, $f_2(\rho)$, and $g(\rho)$ are solutions of the radial GL equations (eqs. 4.24 of ref. 6). The type I component is non-unitary and parameterized by $g$, which is non-vanishing for $\rho < \xi_T$ and vanishes as $\rho^{-2}$ for $\rho \gg \xi_T$.

The non-unitary vortex of eq. (6) has a spin polarization which can be understood qualitatively by expanding the Cooper pair wave function.

$$\langle \uparrow \uparrow \rangle = e^{i\Phi}$$

and writing the amplitudes $a_{m_j}$ in terms of the $A_{\mu \nu}$ in eq. (6): $a_0 = e^{i\Phi}(f_1 + f_2)$, $a_{1\pm} = 0$, and $a_{2\pm} = e^{i\Phi}e^{\pm 2i\Phi}(f_1 + f_2) + 2g$. Thus, the spin polarization

$$\langle \uparrow \downarrow \rangle = |a_{1\pm}|^2 - |a_{2\pm}|^2 \propto g(f_1 - f_2)$$

depends on both the deviation $f_1 - f_2$ of the real part from the axial symmetric state and on the deviation $g$ of the order parameter from unitarity. The vortex spin polarization corresponds to a magnetization $M \sim 10^{11}$ gauss in the region near the vortex core. The detailed structure of these vortices will be discussed elsewhere.

The existence of magnetized vortices may have astrophysical implications. The interiors of rotating neutron stars are believed to consist mostly of $^3P_2$ neutron superfluid, with a small amount of superconducting protons and normal electrons. The $^3P_2$ superfluid must be threaded by an array of quantized vortices in order to participate in the rotation of the star. After a glitch (a discontinuous change in the rotational speed of the star), the conducting fluid transfers angular momentum to the superfluid via electrons scattering off vortices. Feibelman has calculated the relaxation time $\tau_F$ for electrons scattering off excitations in vortex cores, and finds

$$\tau_F = 2.94 \times 10^8 \frac{(p \Delta^2)}{k_F^2} \exp[4.41 \frac{\alpha^2}{k_F^2 T_0}]$$

where $P$ is the pulsar period in seconds, $x$ is the electron number density divided by the neutron number density, $k_F$ the neutron Fermi wavevector in fm$^{-1}$, $\Delta$ the neutron superfluid gap in MeV, and $T_0$ the stellar interior temperature in units of $10^8$K. The strong dependence of $\tau_F$ on both temperature and gap is due to the probability $e^{-\Delta^2/k_F T_0}$ that a neutron excitation is available for scattering. However, electrons can also scatter directly off the vortex core magnetization; for this mechanism, we find

$$\tau_b = 1.26 \times 10^8 \frac{k_F^2 p^{2/3}}{\Delta}$$

This relaxation time is weakly dependent on $\Delta$ and $T$ and limits the velocity relaxation time at low temperatures. For the Crab pulsar ($P = 0.033$ sec and $0.1 < T_0 < 1.0$) we find a day $< \tau < 20$ days where $\tau = (\tau_F^{-1} + \tau_b^{-1})^{-1}$; the observed relaxation times vary from 4.1 to 15 days. The relaxation time in older pulsars increases because they have lower interior temperatures and higher rotational periods (fewer vortices). Since $\tau_b$ limits the relaxation time at low temperatures to 20-100 days (depending on rotational period) glitches in older pulsars, if they occur, may have observable relaxation times.

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