Interaction Effects on the Zeeman Splitting of Collective Modes in Superfluid $^3\text{He}-\text{B}$

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(Received 16 April 1982)

Exact calculations of the $B$-phase collective-mode frequencies in a magnetic field are presented, and they provide new information about the effective mass, the pairing interaction, and the Landau parameters. The calculation of the Zeeman splitting for the $2^+$ mode (real squashing mode) is consistent with the zero-field frequency and agrees quantitatively with the measurements of Avenel et al., for a range of values of $m^*$ which includes some, but not all, of the differing experimental results for this important parameter.

PACS numbers: 67.50.Fi

Superfluid $^3\text{He}$ supports a number of order-parameter collective modes with temperature-dependent frequencies of the form

$$\omega(T)^2 = a \Delta(T)^2 + b (qv_f)^2,$$

where $a$ and $b$ are weakly temperature-dependent functions of order 1. The zero-sound dispersion relation $\omega = \epsilon_0 \Delta$ crosses the dispersion relations for these modes when $q v_f / \Delta(T) - v_f / \epsilon_0 \approx 1$, and hence in discussing their excitation by ultrasound one can generally neglect the dispersion of the collective modes. It is suggestive to think of the collective modes as excited states of the Cooper pairs, and to regard their excitation by zero sound as an ultrasonic Cooper-pair spectroscopy analogous to the optical spectroscopy of diatomic molecules. This analogy is strengthened by studies of zero-sound absorption in a magnetic field. For example, Schopohl and Tewordt\textsuperscript{1-2} predicted that a small magnetic field should produce a linear "Zeeman splitting" of both the real and imaginary $J = 2$ order-parameter modes in $^3\text{He}-\text{B}$ (the $2^+$ and $2^-$ modes, in our previous notation,\textsuperscript{3} which we follow henceforth). The collective modes are then labeled by $m_\nu$, their angular momentum projection along $\hat{H}$, and their frequencies have the form

$$\omega_{\nu}(T, H) = \omega_{\nu}(T) + m_\nu g_{2\nu}(T) \omega_1(T, H),$$

where $\omega_1(T, H)$, the temperature-dependent effective Larmor frequency, is proportional to the magnetic field.

The Zeeman splitting of the $2^-$ mode has yet to be observed, but the splitting of the $2^+$ mode has been measured by Avenel, Varoquaux, and Ebisawa;\textsuperscript{4} their observation of a fivefold splitting of the new ultrasonic absorption line discovered by Giannetta et al.\textsuperscript{5} and Mast et al.\textsuperscript{6} established conclusively that this absorption line is associated with the $2^+$ mode. Equation (2) gives $\delta \omega(T, H)$, the linear splitting of the modes at a fixed temperature, but experimentally one measures $\delta T(\omega, H)$, the splitting in temperature of the modes at a fixed frequency. If we expand Eq. (2) around $T_0$, the temperature of the absorption line in zero field, we obtain

$$\delta T_{2\nu}(\omega, H) = \frac{\omega_{2\nu}(T_0) \omega_1(T_0, H)}{\partial \omega_{2\nu}(T_0)/\partial T}.$$

Schopohl and Tewordt\textsuperscript{7} calculated the Zeeman splittings of the $2^\pm$ modes keeping only the Landau parameter $F_0$ and assuming a pure $l = 1$ pairing interaction. With the help of a new identity given below [Eq. (14)], their results can be written as

$$\delta T_{2\nu}(T) = \frac{1}{H} \left[ 1 - (1 - Y) \lambda(\omega_{2\nu}) \right],$$

$$\omega_{2\nu}(T, H) = \gamma H/\left[ 1 + F_0 g^{(2)}(2, 1/2) \right].$$

The function $\lambda$, first introduced by Tsuneto,\textsuperscript{8} is defined in Ref. 3, and $Y$ is the Yoshida function. However, one cannot consistently compare their results with the measured values of $\delta T_{2\nu}(\omega, H)$, because the frequency $\omega_{2\nu}(T) = (2)^{1/2} \Delta(T)$ obtained with their assumptions does not agree with the measured frequency $\omega_{2\nu}(T)$.

In Ref. 3 we showed that $\omega_{2\nu}(T)$ depends only on $F_0$ and the $l = 3$ pairing interaction $\nu_\nu$; a range of possibilities for $F_0$ and $x_{\nu}^{na} = (\nu_1)^{na} - (\nu_3)^{na}$ between $F_0 = -1.56$, $x_{\nu}^{na} = 0$, and $F_0 = 0$, $x_{\nu}^{na} = -0.432$ all yield results for $\omega_{2\nu}(T)$ in good agreement with experiment.\textsuperscript{4,6} Hence for consistency
it is essential to include $F_s^{\pm \alpha}$ and $\omega_\perp$ in the calculation of $g_{2s}$ and $\omega_\perp$. We have carried out this calculation, which is a lengthy exercise in perturbation theory (details will be provided later). We find that ultrasonic spectroscopy can yield both $F_s^{\pm \alpha}$ and $x_3^{\pm \alpha}$, but only if the effective mass $m^*$ enters $\delta T_{2s}$ implicitly through $F_s^{\pm \alpha}$ is known accurately. However, consistency between the measured values of $\omega_\perp$ and $\delta T_{2s}$ already yields bounds on $m^*$ which include the values of Wheatley and Greywall and Busch, but not those of Alvesalo et al.

Our result for $g_2^+$ is

$$g_2^+=\frac{9}{4} WN_2^+ / D_2^+;$$

$$W=1-\frac{\omega^2}{4\Delta^2};$$

$$N_2^+=(1+\frac{1}{2} F_s^{\pm \alpha} W \lambda)(\lambda+1-Y)+\frac{1}{2} F_s^{\pm \alpha} W \lambda \left[\lambda-1+(1+\frac{1}{2} F_s^{\pm \alpha} \lambda) Y \right];$$

$$D_2^+=2\lambda \frac{\omega^2}{4\Delta^2} \left[1-\frac{5}{3} X_3^{\pm \alpha} W^2 \lambda \right]+\frac{5}{3} \left(\omega \frac{\partial \lambda}{\partial \omega} \right) W \left(\frac{2}{5} - \frac{\omega^2}{4\Delta^2} \right).$$

For $g_2^-$ we find

$$g_2^-=-\frac{2}{5} N_2^- / WD_2^-;$$

$$N_2^-=(\omega^2/4\Delta^2)(\lambda-1+Y),$$

$$D_2^- = 2\lambda \frac{\omega^2}{4\Delta^2} \left[1+\frac{3}{25} F_s^{\pm \alpha} \lambda + \frac{2}{x_3^{\pm \alpha}} \left(\frac{\omega^2}{4\Delta^2}-\frac{1}{2} \lambda \right) + \frac{\partial \lambda}{\partial \omega} \left(\frac{3}{5} - \frac{\omega^2}{4\Delta^2} \right) \right].$$

In all of these expressions, $\omega_\perp$ should be interpreted as the appropriate $H=0$ frequency $\omega_2^\perp(T)$. The effective Larmor frequency is

$$\omega_L = \frac{(1+\frac{1}{2} F_s^{\pm \alpha}) H}{1+F_s^{\pm \alpha} \left(\frac{1}{2} + \frac{1}{3} Y \right) + \frac{1}{2} F_s^{\pm \alpha} \left[\frac{1}{2} + \left(\frac{1}{2} + F_s^{\pm \alpha} \right) Y \right] \lambda}.$$

Hasegawa and Namaizawa have reported results for $\delta \omega(T, H)$ including $F_s^{\pm \alpha}$ but not $\omega_\perp$. For $x_3^{\pm \alpha}=0$ our results agree with those of Hasegawa and Namaizawa except that they are missing the terms proportional to $\partial \alpha / \partial \omega$ in $D_{2s}$, which in general give corrections as large as the other corrections from $F_s^{\pm \alpha}$ and $x_3^{\pm \alpha}$. To obtain our result for $g_2^+$ we have used a new identity,

$$-\Delta^\alpha = \frac{\lambda+1-Y}{4W} \left[1+\frac{1}{2} \frac{d}{d E} \left(\frac{\tanh(E/2T)}{E} \right) \right] \frac{\lambda+1-Y}{4W}.$$

where $E=(\xi^2+\Delta^2)^{1/2}$. With this identity, one easily verifies that our result reduces to that of Schopohl and Tewordt and Avenel, Varoquaux, and Ebisawa when $F_s^{\pm \alpha}=x_3^{\pm \alpha}=0$.

In Fig. 1 we plot $g_{2s}(T)$ for the two extreme cases $F_s^{\pm \alpha}=0$, $x_3^{\pm \alpha}=0$ (hereafter case I) and $F_s^{\pm \alpha}=-1.56$, $x_3^{\pm \alpha}=0$ (hereafter case II) corresponding to $\omega_L(0)=1.075 \Delta_{BCS}(0)$, with $\Delta_{BCS}(T)$ the weak-coupling BCS energy gap. For comparison we also show the "bare" $g_2^+$ factors calculated for $F_s^{\pm \alpha}=x_3^{\pm \alpha}=0$. If we take $\Delta(0)=\Delta_{BCS}(0)$, then for fixed values of $F_s^{\pm \alpha}$, $F_s^{\pm \alpha}$, and $x_3$, $g_{2s}$ and $\omega_\perp$ are universal functions of $T/T_c$, as are $\omega_\perp(0)/\Delta(0)$ and $\partial \omega_\perp/\partial T$. Among these functions only $\omega_\perp$ depends on $F_s^{\pm \alpha}$. In Fig. 2 we show the experimental result of Avenel, Varo-

FIG. 1. $g$ factors for the $J=2\pm$ modes. Calculations of $g_{2s}(T)$ for $\omega_\perp(0)/\Delta_{BCS}(0)=1.075$ are the curves labeled for cases I and II (III and IV). For comparison, $g_{2s}(T)$ calculated without Fermi-liquid or $l=3$ pairing-interaction corrections is the upper (lower) dashed curve.
quaux, and Ebisawa for \( \delta T_{2+}/H \) at \( p = 11 \) bars, together with our theoretical results for \( \delta T_{2+}/H \) in the limiting cases I and II, using both \( F'_{s} = -0.724 \) and \( F'_{s} = -0.804 \), which correspond to the values of \( m^* \) given by Wheatley and Alevsalto et al. respectively. The ranges of possible values of \( \delta T_{2+}/H \) obtained with different values of \( F'_{s} \) do not overlap, and the experimental points fall outside the range of possibilities consistent with the \( m^* \) of Ref. 11. The allowed effective masses lie between \( m^*/m = 3.7 \) and 4.7. As an indication of the uncertainty in \( \delta T_{2+}/H \), we have included a statistical error bar calculated from the data shown in Fig. 2 of Ref. 4. Using Wheatley’s \( m^* \), we find \( F'_{s} = -0.612 \) and \( x_{3}^{-1} = -0.263 \), but these results are obviously sensitive to errors in \( m^* \). We have chosen to plot \( \delta T_{2+}/H \) as a function of \( T_{0}/T_{c} \). We also show on the horizontal axis at the top of Fig. 2 the values of \( \omega_{ACS}(0) = \omega_{ACS}(T_{0})/\Delta_{BCS}(0) \) for the choice of \( F'_{s} \) and \( x_{3}^{-1} \) which corresponds to the theoretical curve (dotted line) through the experimental point. Figure 3 shows the theoretical and experimental results for \( \delta T_{2+}/H \) at \( p = 3.5 \) bars, which again are inconsistent with the effective mass of Ref. 11. The allowed values lie between \( m^*/m = 3.1 \) and 3.8. Using Wheatley’s \( m^* \) (\( F'_{s} = -0.701 \)) we find \( F'_{s} = -0.794 \) and \( x_{3}^{-1} = -0.212 \).

To date the Zeeman splitting of the \( 2- \) modes has not been observed. As Fig. 1 shows, part of the difficulty is that \( g_{2-}(T) \) is much smaller than the \( g \) factor for the \( 2+ \) modes, except very close to \( T_{c} \). Also the interactions which shift the \( 2- \) mode frequency from \( \omega_{2-}(0)/\Delta_{BCS}(0) = \sqrt{2} = 1.41 \) to \( \omega_{2-}(0)/\Delta_{BCS}(0) = 1.40 \) at low pressure further reduce \( g_{2-} \) below the “bare” \( g_{2-} \) calculated by Schopohl and Tewordt. Recently Halperin reported the absence of Zeeman splitting of the \( 2- \) modes in acoustic impedance measurements at \( p = 4.88 \) bars. In a transverse field of 0.345 kG they observed no change in the 10- \( \mu \)W k length of the \( 2- \) mode peak at \( T_{0}/T_{c} = 0.82 \). In Fig. 4 we show \( \delta T_{2-}(T_{0})/H \) calculated for the two extreme cases \( F'_{s} = 0.605 \) and \( F'_{s} = 0 \) (case III) and \( F'_{s} = -1.16 \) and \( F'_{s} = -1.56 \) (case IV) corresponding to \( \omega_{2-}(0)/\Delta_{BCS}(0) \), and for both \( F'_{s} = -0.798 \) and \( F'_{s} = -0.792 \), which correspond to the \( \delta = 4.88 \)-bar values of \( m^* \) from Refs. 9 and 11, respectively. The dotted curve in Fig. 4 corresponds to Wheatley’s \( m^* \) and to \( F'_{s} = -0.794 \) and \( x_{3}^{-1} = -0.212 \), and \( F'_{s} = -0.293 \), which fit the low-pressure values of \( \omega_{2-}(0)/\Delta_{BCS}(0) \).
FIG. 4. Zeeman splitting of the \( J = 2 \) modes at \( p = 4.88 \) bars. Bounds on \( \delta T_{2J}/H \) from cases III and IV. The solid (dashed) curves are \( \delta T_{2J}/H \) calculated for \( F = -0.708 \) \((-0.792)\). The dotted curve is the best estimate of \( \delta T_{2J}/H \) based on interaction parameters determined from the low-pressure measurements of \( \omega_{2\perp}, \delta T_{2\perp}/H, \) and \( \omega_{2\parallel} \).

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\Delta_{BCS}(0), \quad \delta T_{2\perp}/H (\text{Fig. 3}), \quad \omega_{2\perp}(0) = 1.40 \Delta_{BCS}(0).
\]

For \( T_{0}/T_{c} = 0.84 \), \( \delta T_{2\perp}/H = 4.8 \, \mu K/kG \), so that the splitting in temperature between the \( m_{J} = \pm 2 \) modes at \( H = 0.35 \) kG would be \( 6.7 \, \mu K \), which is less than the experimental width of \( 10 \, \mu K \). Even the optimistic estimate of \( \delta T_{2\perp}/H = 6.6 \, \mu K \) would give a maximum splitting of only \( 9.2 \, \mu K \). If the small magnitude of the splitting is the explanation for the negative experimental result for the Zeeman splitting of the \( 2\perp \) modes, then shifting the position of the \( 2\perp \) mode to \( T_{0}/T_{c} = 0.54 \) should resolve the \( M_{J} = \pm 2 \) modes, since their splitting would be \( 16.4 \, \mu K \) in a 0.5-kG field.

We thank W. P. Halperin, J. B. Ketterson, and D. B. Mast for several helpful conversations. This work was supported in part by National Science Foundation Grants No. NSF DMR 802063 and No. NSF DMR 800522.

13We use the precise values of \( T_{0}/T_{c} \) reported to us by E. Varoquaux, private communication.
14There are twelve data shown in Fig. 2 of Ref. 4, from which we calculate \( \delta T_{2\perp}/H = 36.0 \pm 1.0 \, \mu K/kG \), where the uncertainty is the standard deviation of the mean.
15There may also be important systematic errors due to magnetic field and temperature inhomogeneities in the cell and other uncertainties in the thermometry. E. Varoquaux, private communication. However, a 4\% error in the temperature scale would not change our conclusion about \( \mu \alpha^{*} \).
17W. P. Halperin, to be published.