JOSEPHSON TUNNELING BETWEEN SUPERCONDUCTORS WITH DIFFERENT SPIN AND SPACE SYMMETRIES*

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ABSTRACT

We argue that Josephson tunneling between a conventional s-wave superconductor and an odd-parity (triplet) superconductor, with the conventional frequency $2eV/h$, is possible even when the boundary, represented by a tunneling Hamiltonian, is time-reversal symmetric. Thus, an observation or non-observation, of Josephson tunneling with frequency $2eV/\hbar$ is not a strict criterion for singlet or triplet superconductivity. This result is relevant for heavy fermion superconductors in which spin-orbit scattering is of central importance.

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The discoveries of superconductivity in the class of heavy fermion metals, $CeCu_2Si_2$, UBe_{13}, etc.,/1/ has revived earlier interests in possible spin-triplet superconductors. Anderson,/2/ and Valls and Tesanovič /3/ have suggested that heavy fermion compounds exhibiting large spin fluctuation effects and large mass enhancements bear a close resemblance to liquid ^3He, and thus may condense into a spin-triplet superconducting state. This possibility was emphasized (for UBe_{13}) by Ott *et.al.*,/4/ whose specific heat measurements show $C \sim T^3$ for $T < T_c$. On this basis, and theoretical arguments similar to those of Anderson, Ott *et.al.* concluded that superconductivity in UBe_{13} is similar to the p-wave ABM state in superfluid ^3He-A, because the energy gap for the ABM state vanishes at points on the Fermi surface giving $C \sim T(T/T_c)^2$ for the low temperature specific heat. Similarly, Bishop *et.al.* measured the ultrasound attenuation in UPt_3, and concluded that observed power law behavior $(\alpha_5/\alpha_{||}) \sim T^2$ was strong evidence that this system is a p-wave state with a line of zeroes on the Fermi surface, analogous to the polar phase in isotropic p-wave superfluids. This conclusion suports an earlier suggestion by Varma /6/ that superconductivity in UPt_3 is in a p-wave polar state. This conclusion has been challenged by Rodriquez,/7/ who noted that in the hydrodynamic limit the ultrasound attenuation in UPt_3 is consistent with an ABM-like state with point zeroes of the energy gap, provided that the mean free path is energy independent. More recently Miyake, *et.al.* /8/ have argued that the pairing interaction for heavy fermion quasiparticles is dominated by the exchange of antiferromagnetic spin fluctuations that favor unconventional even-parity superconductivity.

However, conclusions regarding the spin symmetry and parity of the superconducting state, based on power law behavior of the thermodynamic and transport properties, are ambiguous. Superconducting order parameters of either spin symmetry (S=0 or S=1) can give rise to power law behavior of the specific heat and ultrasound. In addition, Anderson, /9/ Blount, /10/ and Volovik and Gorkov /11/ have pointed out that the allowed symmetry classes of the superconducting order parameters, consistent with strong spin-orbit coupling and the relevant point symmetries of the heavy fermion superconductors, do not allow S=1 order parameters with lines of nodes on the Fermi surface, thus, also calling into question the polar phase interpretation of UPt_3. The bottom line is that it is difficult to definitively deduce the spin symmetry, or parity, of the superconducting state based on the above comparisons.

Pals *et.al.* /12/ have suggested that Josephson tunneling experiments can provide a 'yes' or 'no' answer to the question 'what is the spin symmetry and parity of a gien supercondutor?' The argument is that an AC Josephson effect with frequency $2eV/\hbar$ does not exist betwen a conventional s-wave (S=0) superconductor and an odd-parity (S=1) superconductor, at least not if the boundary is time-reversal invariant. Several authors have used this criterion to test whether or not a candidate triplet superconductor does in fact have an odd-parity order parameter. Recently, Noer *et.al.* /13/ have observed an AC Josephson effect in a $Nb--Lu_2Fe_3Si_5$ point contact junction. Since Nb is a conventional S=0 superconductor, these authors concluded that $Lu_2Fe_3Si_5$ is not a triplet superconductor in spite of the correlation between superconductivity and the existence of the magnetic Fe ions in this system and other circumstantial evidence consistent with odd-parity superconductivity. similarly, Steglich *et.al.* /14/ have reported a DC Josephson effect between Al and $CeCu_2Si_2$ and concluded that $CeCu_2Si_2$ is a singlet heavy fermion superconductor. We show below that the conclusion by Pals *et.al.* is not generally valid, even when the boundary Hamiltonian is time reversal symmetric, and thus an observation, or non-observation, of Josephson tunneling with frequency $2eV/\hbar$ is not a strict criterion for singlet or triplet superconductivity. Fenton /15/ has previously noted that strong spin-orbit coupling in heavy fermion metals, combined with spatial variations of the triplet order parameter near the boundary, can lead to a significant, albeit small (*i.e.,* reduced by a factor of atomic size divided by Cooper-pair size, relative to conventional Josephson currents), Josephson coupling between a singlet superconductor and a presumed triplet superconductor. Our argument differs qualitatively from that of Fenton, and in the general case of triplet superconductivity in metals with strong spin-orbit coupling leads to a Josephson current of the conventional magnitude.

The superconducting order paramter, or electron Cooper-pair amplitude $\Delta_{\alpha\beta}(\vec{k}) =< a_{\vec{k}\alpha}\, a_{-\vec{k}\beta} >$ is totally antisymmetric.

$$\Delta_{\alpha\beta}(\vec{k}) = -\Delta_{\alpha\beta}(-\vec{k}). \tag{1}$$

In metallic systems with inversion symmetry, which includes all of the heavy fermion superconductors of current interest, it is reasonable to assume that the superconducting order parameter will retain this symmetry in which case $\Delta_{\alpha\beta}$ is either even parity (spin singlet) or odd parity (spin triplet). The corresponding pair amplitudes $\Delta_{\alpha\beta}(\vec{k})$ may be represented.

$$\begin{aligned} \Delta_{\alpha\beta} &= (i\sigma_2)_{\alpha\beta} \cdot \Delta_0(\vec{k}) \qquad (S=0), \\ \Delta_{\alpha\beta}(\vec{k}) &= (i\vec{\sigma}\,\sigma^2)_{\alpha\beta} \cdot \vec{\Delta}(\vec{k}) \qquad (S=1), \end{aligned} \tag{2}$$

where $(i\sigma_2)$ and $(i\sigma_j\,\sigma_2|j = 1,2,3)$ are the antisymmetric and symmetric Pauli matrices. For metals in which the spin-orbit interaction is important, in particular, heavy fermion metals, spin is not a good quantum number, but as Anderson /8/ pointed out the order parameter may still be classified as *singlet* or *triplet* with respect to a pseudo-spin space provided the superconducting state does not destroy inversion symmetry. We use the generic term *spin* to refer to the relevant pseudo-spin space in a given metal.

Consider a junction between two different metals. The barrier between the metals is represented by a transfer Hamiltonian.

$$H = \sum_{k,q} \Big\{ t_{k,q} \;\; a_k^+\, b_q + t_{k,q}^* \;\; b_q^+\, a_k \Big\}, \tag{3}$$

where a_k^+, (b_q^+) creates an electron in state $k\,(q)$ on the left (right) of the barrier, and $t_{k,q}$ measures the energy in transfering a single electron across the barrier. The labels k and q represent both momentum and spin of the electron states.

Thus, $t_{k,q}$ is A 2×2 spin matrix function of $(\vec{k}, \vec{q})$, *i.e.*, $t_{k,q} = t(\vec{k}, \vec{q})_{\alpha\beta}$. This transfer Hamiltonian generalizes the spin-conserving transfer Hamiltonian used by Pals *et.al.*. The transfer matrix (t-matrix) may be separated as $t(\vec{k}, \vec{q}) = t_0\, 1 + \vec{t} \cdot \vec{\sigma}$; the conventional spin conserving part of the t-matrix is t_0, while $\vec{t}$ determines the spin-dependent electron transfer accross the barrier. In junctions where at least one electrode is a heavy fermion metal the spin-dependent contribution, $\vec{t}$, will be non-vanishing and of the same order of magnitude as the spin-independent amplitude t_0, even if the boundary potential is not intrinsically magnetic; since the pseudo-spin representations for different electrodes are in general different the t-matrix will not be diagonal in the product space formed from the left and right spin states.

The tunneling current is given in equations (4-5) below to leading order in the tunneling Hamiltonian. This calculation parallels that of Ambegaokar and Baratoff, /16/ so we simply state the results. The total tunneling current to second order in H separates into a single particle tunneling contribution, I_N, and the Josephson pair current I_S. If a DC voltage V exists between the two superconductors the quasiparticle and pair currents are

$$I_N = 2e\, N_{OL}\, N_{OR}\, I_m \left\{ \int \frac{d^2k}{4\pi} \int \frac{d^2q}{4\pi} \sum_{\alpha\alpha'\beta\beta'} t(\vec{k}, \vec{q})_{\alpha\beta}\; t^*(\vec{k}, \vec{q})_{\alpha'\beta'} \right. \\ \left. K^N(\vec{k}, \vec{q}; i\omega_n u \to \frac{eV}{\hbar} + io)_{\alpha\alpha', \beta\beta'} \right. , \tag{4}$$

$$I_S = 2e\, N_{OL}\, N_{OR}\, I_m \left\{ e^{2i\,eVt/\hbar} \int \frac{d^2k}{4\pi} \int \frac{d^2q}{4\pi} \sum_{\substack{\alpha\alpha' \\ \beta\beta'}} t(\vec{k}, \vec{q})_{\alpha\beta}\;\; t(-\vec{k}, -\vec{q})_{\alpha'\beta'} \right. \\ \left. K^S(\vec{k}, \vec{q}; i\omega_\nu \to -\frac{eV}{\hbar} + io)_{\alpha\alpha'\beta\beta'} \right. , \tag{5}$$

where $K^{N,S}$ are given in terms of the thermal Green's functions for the left (L) and right (R) superconductors.

$$K^N = -1/\beta \sum_{\nu'} \overline{g}_L(\vec{k}, \varepsilon_{\nu'})_{\alpha\alpha'}\;\; g_R(\vec{q}, \omega_\nu - \varepsilon_{\nu'})_{\beta\beta'} \;, \\ K^S = 1/\beta \sum_{\nu'} \overline{f}_L(\vec{k}, \varepsilon_{\nu'})_{\alpha\alpha'}\;\; f_R(\vec{q}, \omega_\nu - \varepsilon_{\nu'})_{\beta\beta'} . \tag{6}$$

The Green's functions which appear in these equations are the quasiclassical Green's functions integrated with respect to $|\vec{k}|$ or $|\vec{q}|$, and are the solutions of Eilenberger's equation of motion and normalization condition. Thus, the variables $\vec{k}$ and $\vec{q}$ are the momenta for the left and right superconductors evaluated on their respective

Fermi surfaces. The uniform bulk solutions are given by

$$\begin{aligned} g\,(\vec{k},\varepsilon_\nu) &= \pi \frac{i\,\varepsilon_\nu}{\sqrt{\varepsilon_\nu^2 + |\Delta(\vec{k})|^2}}\; 1_{spin}; \qquad \overline{g}\,(\vec{k},\varepsilon_\nu) = -g\,(\vec{k},\varepsilon_\nu), \\ f\,(\vec{k},\varepsilon_\nu) &= -\pi \frac{\Delta\,(\vec{k})}{\sqrt{\varepsilon_\nu^2 + |\Delta\,(\vec{k})|^2}}; \qquad \overline{f}\,(\vec{k},\varepsilon_\nu) = -f\,(\vec{k},\varepsilon_\nu)^+ \ , \end{aligned} \tag{7}$$

These solutions are valid for either singlet or triplet order parameters; however their applicability in the case of triplet superconductors is restricgted to unitary states with

$$\Delta\Delta^+ = |\Delta|^2 \ \ 1_{spin} \ . \tag{8}$$

or, in the case of nonunitary triplet states, to $\frac{T<}{\sim T_c}$. Finally, we note that the prefactors N_{OL} and N_{OR} in equations (4-5) are the densities of states at the Fermi surface in the normal state for the L and R superconductors, and the integrations are taken over the fermi surfaces of the two metals.

If both metals are normal (*i.e.* above their transition temperatures) the quasi-particle current reduces to Ohm's law with a junction conductance.

$$1/R_N = 4\pi e^2 \ N_{0L} \ N_{0R} \ < |t_0|^2 + |\vec{t}|^2 >, \tag{9}$$

where $< \ldots >= \int \frac{d^2k}{4\pi} \int \frac{d^2q}{4\pi} (\ldots)$.

The classic result for a tunnel junction separating two identical singlet s-wave superconductors can be obtained, with spin-dependent tunneling across the barrier, provided the barrier is time-reversal invariant, *i.e.* $H = \Theta^{-1} \, H \, \Theta$ where Θ is the antiunitary time-reversal operator whose action on the electron fields is

$$\Theta^{-1} \, a_{\vec{k}\alpha} \, \Theta = (i\sigma_2)_{\alpha\beta} \ a_{-\vec{k}\beta} \tag{10}$$

Time reversal invariance of the barrier Hamiltonian is equivalent to the following conditions on the t-matrix.

$$\begin{aligned} t_0\,(\vec{k},\vec{q})^* &= t_0\,(-\vec{k},-\vec{q}), \\ \vec{t}\,(\vec{k},\vec{q})^* &= -\vec{t}\,(-\vec{k},-\vec{q})\,. \end{aligned} \tag{11}$$

Using these equations to evaluate the Josephson current for identical s-wave superconducting electrodes gives

$$I_S = \frac{\pi\Delta}{2eR_N} \ \tanh\,(\beta\Delta/2) \ \sin\,(\phi + \frac{2eV}{\hbar}\,t) \ , \tag{12}$$

where ϕ is the static phase difference between L and R order parameters, which is the result of Ambegaokar and Baratoff. The above result includes time-reversal invariant spin-orbit scattering in R_N.

The particularly interesting case is tunneling from a singlet superconductor to a triplet superconductor. For simplicity consider the special case where the temperature is close to both transition temperatues. In this situation,

$$f_R = -\pi \Delta_R(\vec{q})/|\varepsilon_\nu| \quad ; \quad \Delta_R = i\vec{\sigma}\sigma_2 \cdot \vec{\Delta}(\vec{q}),$$

$$\overline{f}_L = -\pi \overline{\Delta}_L(\vec{k})/|\varepsilon_\nu| \quad ; \quad \overline{\Delta}_L = -i\sigma_2\, \Delta_0^*(\vec{k}) \; .$$

and the Josephson current reduces to

$$\begin{aligned} I_S &= -\pi^2 N_{OL}\, N_{OR}\, I_m \left\{ e^{2ieVt/\hbar} < \Delta_0^*(\vec{k})\, \vec{\Delta}(\vec{q}) \cdot \vec{\omega}(\vec{k},\vec{q}) > \right\}, \\ \vec{\omega} &= 2Re\left\{ t_0(\vec{k},\vec{q}) \vec{t}^*(\vec{k},\vec{q}) + i\vec{t}^*(\vec{k},\vec{q}) \times \vec{t}(\vec{k},\vec{q}) \right\}, \end{aligned} \tag{14}$$

where we have used time-reversal symmetry to simplify the equation for $\vec{\omega}$. If we neglect the spin-flip contribution to the t-matrix we obtain the result of Pals *et.al.* We emphasize here that when $\vec{\omega} = 0$ the supercurrent vanishes even if (i) the t-matrix breaks time-reversal symmetry, or (ii) the bulk triplet order parameter is non-unitary, implying that the triplet superconducting state breaks time-reversal symmetry. A more subtle case may be possible in which the triplet state breaks time-reversal symmetry locally near the interface and generates a Josephson coupling.

The spin-dependent t-matrix will contribute to the tunneling current if the barrier provides a significant spin-orbit potential For the electrons. This case is most likely in tunnel junctions where one electrode contains heavy elements. Junctions between a heavy metal, containing rare earth ions, and a lighter metal are good candidates for tunneling via the spin-orbit interaction. For barriers separating dissimilar metals, one of which is a heavy fermion metal, we expect $\vec{\omega} \neq 0$. In particular, for a time-reversal symmetric, translationaly invariant interface the t-matrix depends only on the conserved projection of the momentum, $\vec{k}_{||}$, in the plane of the barrier, and the normal to the interface $\hat{n}$. And if the interface is symmetric with respect to reflections through any plane perpendicular to the interface then $\vec{t} = t_{SO}\,(\hat{n}\, x\, \vec{k}_{||})$, in which case the supercurrent is proportional to

$$I_S \sim I_m\left\{ e^{2ieVt/\hbar} < Re\,(t_0\, t_{so}^*)\; \Delta_0^*(\vec{k})\; \vec{\Delta}(\vec{q}) \cdot (\hat{n} \times \vec{k}_{||}) > \right\} \tag{15}$$

Thus, although time-reversal symmetry may prevail in the barrier in the absence of interface magnetism, spin-orbit scattering can lead to a non-vanishing Josephson current, with frequency $2eV/\hbar$, between singlet and triplet superconductors. We note that the condition $\vec{\omega} \neq 0$, and thus a necessary condition for singlet-triplet Josephson tunneling, is that the full Hamiltonian break inversion symmetry. This will clearly be the case for two very dissimilar metals connected by a tunneling barrier. However, the current will in general be sensitive to the orientation of the triplet order parameter relative to the barrier. Finally, we note that, as is known in superfluid ^3He, that boundaries represent strong perturbations which deform the triplet order parameter, leading to new superfluid phases in a region of size determined by the coherence length./17/ Any quantitative theory of Josephson tunneling involving a triplet supercondutor (or any non-s-wave superconductor) requires knowledge of the perturbed region of superconductivity near the boundary, suggesting that Josephson tunneling in heavy fermion metals will also be sensitive to the internal structure of the interface.

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