Comment on "Symmetry and structure of quantized vortices in superfluid $^3$He-B"

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Recent theoretical attempts to explain the observed vortex-core phase transition in superfluid $^3$He-B yield conflicting results. Variational calculations by Fetter and Theodorakis, based on realistic strong-coupling parameters, yield a phase transition in the Ginzburg-Landau region that is in qualitative agreement with the phase diagram. Numerical precise calculations by Salomaa and Volovik (SV), based on the Brinkman-Serene-Anderson (BSA) parameters, do not yield a phase transition between axially symmetric vortices. The ambiguity of these results is in part due to the large differences between the $\beta$ parameters, which are inputs to the vortex free-energy functional. We comment on the relative merits of the $\beta$ parameters based on recent improvements in the quasiparticle scattering amplitude and the BSA parameters used by SV.

In a recent paper on the symmetry and structure of quantized vortices in superfluid $^3$He-B, Salomaa and Volovik (SV) discuss the possibility of a vortex-core phase transition in the Ginzburg-Landau (GL) region of the phase diagram. They argue that it is difficult to predict a vortex-core transition because the differences in energy between different vortices are small; hence precise numerical procedures are necessary to obtain reliable results. SV criticize recent calculations by Fetter and Theodorakis (FT) on precisely this point. FT adjust the variational parameters of their trial solutions for the vortex order parameter to minimize the GL free-energy functional. Using the strong-coupling GL coefficients from our paper, they then study the pressure dependence of their optimized variational solutions and find a weakly first-order (perhaps second-order) vortex-core phase transition at about 10 bars. However, SV find no evidence for such a phase transition, and suggest that the variational procedure of FT is numerically crude and that the prediction by FT is suspect.

There is, however, another important difference between the variational calculations of FT and the numerical minimization of SV, namely, the input to the GL free-energy functional. All vortex structure calculations in the GL region require the set of five GL coefficients (the $\beta$ parameters referred to by SV) as input. Reasonably accurate $\beta$ parameters are necessary if one wants to predict the correct vortex structure because strong-coupling corrections are important in $^3$He. SV use the Brinkman-Serene-Anderson (BSA) spin-fluctuation theory, which is a one-parameter model that yields qualitatively different $\beta$ parameters as a function of pressure than those used by FT. The $\beta$ parameters used by FT are from our paper and are based on recent improvements in the quasiparticle scattering amplitude (QSA). This important distinction is apparently unappreciated by SV, since they argue that the FT result is further suspect by stating, "The properties of the Sauls-Serene parametrization of... the $\beta$ parameters... yield a phase transition even in bulk $^3$He from the $B$ phase to the $A$ phase at too low a pressure. Certainly one should not calculate the vortex-core structure in the $B$ phase with $\beta$ values corresponding to a stable $A$ phase..." SV mis-state our published results; they also do not provide a reference. Our parametrization does not yield an $A-B$ transition below the polycritical point of 22 bars. Our original conclusion was "...a precise determination of the polycritical point (PCP) is difficult. We have calculated the $\Delta \beta$ at lower pressures... (and) obtain 27 bars for the PCP, compared to the experimental value of 22 bars." Moreover, SV imply that our results for the $\beta$ parameters should not be taken seriously, at least not any more seriously, than the BSA parametrization they use. This is not the case. The BSA $\beta$ parameters cannot account for the magnitudes of the specific-heat jumps or the phase diagram of $^3$He-$A_1$. If the BSA parameter $\delta$ is fit to the $B$-phase specific-heat jump of 1.85 at $p = 29$ bars one obtains $\delta = 1.15$ and an $A$-phase specific-heat jump of $\Delta C_p^{BA} = 3.0$, in serious disagreement with the experimental value of 1.95. Recent measurements of the $^3$He-$A_1$ phase diagram yield experimental values for the ratio $R = -\beta_3/\beta_{245}$. At $p = 29$ bars $R^{exp} = 1.6$, while the BSA ratio is $R^{BSA} = 3.5$. Thus, the BSA $\beta$ parameters do not accurately reflect the relative weights of strong-coupling corrections to different thermodynamic quantities. Neither can the BSA parameters represent the pressure dependence of strong-coupling corrections. If $\delta$ is determined at lower pressures from the $B$-phase specific-heat jump, then we find that the BSA theory predicts the $A$ phase to be stable down to a pressure of 10 bars. BSA also predicts the ratios of the strong-coupling corrections to be independent of pressure. For example, $(\Delta \beta_3/\Delta \beta_2)_{BSA} = -0.29$ for all pressures. In contrast, our result for this ratio, calculated from near-perfect fits of the QSA to the normal Fermi-liquid properties, varies from $+0.22$ ($p = 34$ bars) to $+0.41$ ($p = 12$ bars). The BSA parameters yield poor results because they are based on one-parameter spin-fluctuation theory. If the QSA for this theory is adjusted to give reasonable agreement with the normal-state transport scattering time, then the calculated effective mass is too small by more than a factor of 2.

Recently, several models for the QSA which accurately reproduce the normal-state transport coefficients and Landau parameters have been proposed and refined. Levin and Valls have recently reviewed the subject. Our main point is that calculations of the strong-coupling $\beta$ parameters based on such a QSA give good agreement with the measured specific-heat data. Our results for the $\beta$ parameters correspond to specific-heat jumps for the $A$ and..."
$B$ phases that are within 12% of experimental results.\textsuperscript{7} Calculations of the strong-coupling coefficients by Bedell,\textsuperscript{14} using a QSA derived from the Aldrich-Pines polarization potentials,\textsuperscript{12} accurately reproduce the specific-heat jumps over the full pressure range. Recent refinements of the QSA by Pfitzner and Wölflé\textsuperscript{15} may do even better. The exceptional set of $\beta$ parameters are the BSA values used by SV.

Our opinion is that exact numerical solutions in the GL theory give little reliable information on the vortex-core structure in $^3$He-$B$ without reasonably accurate $\beta$ parameters. Thus, the final answer to whether or not there is a vortex-core transition between axially symmetric vortices in the GL theory will presumably have to wait until precise numerical solutions with physically accurate inputs are reported.

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\textsuperscript{5}W. F. Brinkman, J. W. Serene, and P. W. Anderson, Phys. Rev. A 10, 2386 (1974). The BSA $\beta$ parameters are given by SV in Eqs. (4.3) of Ref. 1. Unfortunately, $\beta_1$ is given incorrectly. The correct BSA result is $\beta_1 = -(1+0.18)\beta_0$. See also P. W. Anderson and W. F. Brinkman, in The Physics of Liquid and Solid Helium, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), Vol. II, p. 239.
\textsuperscript{6}The correct reference is Ref. 3 above.
\textsuperscript{14}K. Bedell, Phys. Rev. B 26, 3747 (1982).