PROXIMITY EFFECTS BETWEEN CONVENTIONAL AND UNCONVENTIONAL SUPERCONDUCTORS

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In this article I review recent theoretical work on the possibility of using tunneling and proximity effect experiments to study unconventional superconductors. The basic idea is simple: as shown in Fig. 1 one places a thin layer of some well understood conventional superconductor in good metallic contact with the superconductor one wishes to study, and then measures (e.g. by tunneling at the outer edge the conventional layer) how the superconducting properties of the conventional material are altered by its proximity to the unconventional material.

To study this question theoretically one has to solve the gap equation for the inhomogeneous system. Because the symmetry and the physical origin of the pairing interaction may be different for conventional than for unconventional superconductors, it is possible that the proximity effect is different between two conventional superconductors than it is between a conventional and an unconventional superconductor. Our understanding of the nature and observability of these differences is still preliminary. A useful theoretical technique for calculating such proximity effects has only recently been proposed, and only a few calculations in simple model systems have been done. The results from this
preliminary work are not encouraging: proximity effects involving unconventional superconductivity seem to differ only in subtle ways from those involving only conventional superconductivity; further, proximity effect experiments do not seem to provide a useful method of distinguishing unconventional singlet (e.g. $d$-wave) from triplet superconductivity.

The rest of this article is organized as follows: in section II the relevant physics of the proximity effect and of unconventional superconductivity is outlined, and the important problems are posed. In section III some theoretical approaches to the problem are outlined and a few results for an $s$-wave superconductor in proximity to model $d$-wave or $p$-wave superconductors are presented. Josephson coupling between conventional and unconventional superconductors is also briefly discussed. There is a brief conclusion.

This article is a review of the physics of proximity effects involving unconventional superconductors. The emphasis is on the important physics; the reader is directed elsewhere for calculational details.

II

In this section the physics of proximity effects involving anisotropic superconductivity is qualitatively discussed. The mathematical formulation of the problem is given in the next section.

We begin with the proximity effect. Study of the proximity effect concerns itself with the question of how the superconducting properties of one material (e.g. transition temperature, magnitude of gap, penetration depth...) are affected when it is placed in good electrical contact with another superconductor or non-superconducting material as shown e.g. in Fig. 1. Proximity effects involving two conventional $s$-wave superconductors have been studied for many years. For a review see Refs. 1,2. The basic idea behind the proximity effect is simple: if two metals are in good electrical contact, electrons from one material may pass into the other. Thus Cooper pairs from a superconductor may leak into a normal metal, imparting some superconducting properties to a non-superconductor near an interface. Conversely, normal electrons may leak into a superconductor, weakening the superconductivity near the interface.

1). Schematic of physical situation considered in this paper. “A” is a thin layer of metal deposited in good electrical contact upon material B, a superconductor to be studied. Coordinates used in section III are also given.
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The rest of this article is organized as follows: in section II the relevant physics of the proximity effect and of unconventional superconductivity is outlined, and the important problems are posed. In section III some theoretical approaches to the problem are outlined and a few results for an s-wave superconductor in proximity to a model d-wave or p-wave superconductors are presented. Josephson coupling between conventional and unconventional superconductors is also briefly discussed. There is a brief conclusion.

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![Schematic of physical situation considered in this paper.](image_url)

From this physical picture one immediately sees that there are four important parameters governing the proximity effect. These are the bulk superconducting pairing potential and pairbreaking parameters for each of the two materials, the transparency of the interface, and a characteristic length $l_\xi$ for each material.

This length, $l_\xi$, may be defined roughly as follows: the superconducting properties of electrons within a distance $l_\xi$ of the interface are directly influenced by the presence of the interface and of any material beyond the interface. The superconducting properties of electrons farther than $l_\xi$ from the interface are, in general, not. We shall make this definition more precise below. The length $l_\xi$ is usually approximately given by the BCS coherence length: e.g. for a clean material $l_\xi = \sqrt{\hbar V_F / \Delta}$ (here $V_F$ is the Fermi velocity, $T$ is the temperature and $\Delta$ the BCS gap. We use units such that $\hbar = k_B = 1$). Typically $l_\xi = 10^{-10} \text{A}$, much larger than a typical atomic dimension; this considerably simplifies the theoretical and experimental problem. Note that it is only if the interface between the two materials is sharp on the scale of $l_\xi$ that one may parameterize the interface by a transmissivity. Clearly, to maximize the observability of any proximity effect one would like to study films of thickness $d \ll l_\xi$, but one would like $d \gg a$, where a is some atomic length (such as the size of a unit cell or a scale determining the sharpness of the interface) so that the intrinsic pairing interaction etc. in the film is the same as in a bulk sample of the same material.

Results that would be obtained from ideal proximity effect experiments involving conventional s-wave superconductivity are sketched in Fig. 2. The system to which these sketches refer is shown in Fig. I. In Fig. 2a the transition temperature, $T_{cs}$ of the A-B system is shown as a function of the thickness, $d$, of material A, for two cases. Curve (1) is for $T_{cs} = 0$, and curve (2) is for $T_{cs} > T_{cs} > 0$. Here $T_{cs} = 0$ are the bulk transition temperatures of materials A and B respectively. It is assumed that material B is infinitely thick. In Fig. 2b the superconducting order parameter $\Delta$ is plotted as a function of position $x$ (in the direction normal to the A-B interface) for $T_{cs} > T > T_{cs}$. It is assumed that material B is infinitely thick. The length over which $\Delta$ varies is set by $l_\xi$. For $T > 0$, the discontinuity in $\Delta$ is set by the transmissivity of the interface. In Fig. 2a it is assumed that the interface is not pairbreaking (containing, e.g. no magnetic impurities). Then the only role played by the interface is to control the degree to which e.g. the properties of one side influence electrons on the other side near the interface.

The curves in Fig. 2 summarize some of the simplest results of the theory of the proximity effect in conventional superconductivity. From them it is seen that the transmissivity of the interface and the length scale $l_\xi$ merely fix the relevant length scales and the overall magnitude of the various effects. The important physics is contained in the pairing potentials and pairbreaking parameters of the two materials, which determine the superconducting $T_{cs}$. We now consider how these results change when unconventional superconductivity is involved.

We first define "unconventional superconductivity" and outline some of its properties. Further details and references may be found in Ref. 5 and in the heavy fermion literature. A homogeneous superconductor is characterized by an order parameter $\Delta_0(\varepsilon)$ which depends on wave-vector and on two spin variables. The order parameter may be thought of as a bound pair of electrons; $\varepsilon$ is then the Fourier transform of the coordinate describing the relative $|\varepsilon - \varepsilon_j|$ motion, and a
2. Results of proximity effect experiments

a) \( T_A \) of A-B system (measured at outer layer of A) for the two cases (1) material B an s-wave superconductor (2) material B a pair-maker.
b) Gap as function of position for the case \( T_B > T > T_C \). It is the thesis of this paper that the curves in 2a and the curve in Fig. 2b apply to the case B=anisotropic ("d-wave" or triplet) superconductor. Only the magnitude of \( \Delta \) or the discontinuity in \( \Delta \) distinguish anisotropic from conventional superconductors.

and \( \beta \) are the spins. By the Pauli principle \( \Delta \) must change sign if we interchange \( \alpha \) and \( \beta \) and set \( \mathbf{E} \rightarrow -\mathbf{E} \). There are thus two possibilities: singlet, in which \( \Delta \) is odd under interchange of \( \alpha \) and \( \beta \) and even under \( \mathbf{E} \rightarrow -\mathbf{E} \) and triplet, in which the reverse is true. In materials with strong spin-orbit coupling (such as the heavy fermion materials), spin is not a good quantum number; however, it has been shown by Volovik and Gorkov\(^4\) and by Blount\(^5\) that provided one restricts attention to crystals with inversion symmetry, it is possible to define a "pseudospin" index which is conserved as the electron propagates through the crystal. Pseudospin is related to the conventional spin by a rotation which depends on position on the Fermi surface and thus couples to the magnetic field in a complicated way. But as long as one does not consider magnetic fields, one can, for most purposes, treat pseudospin as ordinary spin.

Thus, a superconducting order parameter may be characterized by its parity under spatial inversion, \( \mathbf{E} \rightarrow -\mathbf{E} \). Even parity corresponds to spin singlet, odd to spin triplet. One may further characterize a superconducting order parameter by the way it transforms under operations of the crystal symmetry group. If the order parameter transforms into itself under all rotations of the crystal group, one has "s-wave" (and thus, necessarily, singlet) superconductivity, if not, one has "higher partial wave" "anisotropic" or "unconventional" superconductivity. In both s-wave and anisotropic superconductivity one may have an energy gap which vanishes
and $\beta$ are the spins. By the Pauli principle $\Delta$ must change sign if we interchange $\alpha$ and $\beta$ and set $E_{\alpha} = E$. There are then two possibilities: singlet in which $\Delta$ is odd under interchange of $\alpha$ and $\beta$ and even under $E_{\alpha} = E$ and triplet, in which the reverse is true. In materials with strong spin-orbit coupling (such as the heavy fermion materials), spin is not a good quantum number; however it has been shown by Volokhin and Gorkov\(^5\) and by Blount\(^6\) that provided one restricts attention to crystals with inversion symmetry, it is possible to define a "pseudospin" index which is conserved as the electron propagates through the crystal. Pseudospin is related to the conventional spin by a rotation which depends on position on the Fermi surface and thus couples to the magnetic field in a complicated way. But as long as one does not consider magnetic fields, one can, for most purposes, treat pseudospin as ordinary spin. Thus, a superconducting order parameter may be characterized by its parity under spatial inversion, $E_{\alpha} = E$. Even parity corresponds to spin singlet, odd to spin triplet. One may further characterize a superconducting order parameter by the way it transforms under operations of the crystal symmetry group. If the order parameter transforms into itself under all rotations of the crystal group, one has "s-wave" (and thus, necessarily, single) superconductivity, if not, one has "higher partial wave" "anisotropic" or "unconventional" superconductivity. In both s-wave and anisotropic superconductivity one may have an energy gap which varies

nowhere on the Fermi surface, or on points or un the case of single superconductivity only lines on the Fermi surface.

In an anisotropic superconductor the order parameter must vary in a particular way across the Fermi surface. Therefore, any process which does not conserve crystal momentum can be pairbreaking for such a superconductor. For example, the standard techniques for computing the effect of non-magnetic impurities on superconductivity can be used to show that impurity scattering is pairbreaking for higher partial wave superconductors.\(^9\) The pairbreaking parameter is of order $(1/T_{c_{0}})$, where $T_{c_{0}}$ is the transition temperature of a pure crystal and of the usual impurity scattering time.

Now consider an inhomogeneous situation, e.g. that shown in Fig. 1. The superconducting order parameters will then vary in space, and one must write $\Delta_{\alpha}(\mathbf{r}, \mathbf{E})$. The coordinate $\mathbf{r}$ gives the spatial variation of the order parameter and may be thought of as the center of mass of a Cooper pair. The characteristic scale for variations with $\mathbf{r}$ is the length $\xi$, mentioned above. It is easy to show\(^7\) that up to terms of relative order $1/\xi$, the classification of $\Delta$ in terms of symmetry under rotations of spin and $E$ applies also to the inhomogeneous case: for this purpose $\mathbf{r}$ is a dummy variable.

Note that the scattering (reflection from or transmission through) a boundary can change an electron's momentum or spin, and may therefore be pairbreaking for a non-s-wave superconductor. Note especially that even an interface with no spin-orbit scatterers or magnetic impurities may have a large amplitude for flipping the "pseudo-spin" of an electron if it connects materials with very different spin-orbit couplings. This pairbreaking effect will be different for different possible anisotropic states and for different orientations of the order parameter in a given state. Thus a boundary will tend to orient and also to change the form of a superconducting order parameter. This effect has long been known in the context of the Heisenberg model,\(^11\) but the complications induced by a non-spherical Fermi surface and by spin-orbit coupling have not so far been considered.

The presence of an interface has another peculiar effect. Essentially because electronic momentum and (pseudo-)spin need not be conserved in interaction with an interface, it is possible in general for an interface to convert e.g. an s-wave spin singlet Cooper pair to a d-wave spin singlet or into a p-wave spin-triplet Cooper pair.\(^12\),\(^14\),\(^17\)

The result of these considerations is that in general proximity effects involving unconventional superconductivity are more complicated than proximity effects involving only s-wave superconductivity, but there are no qualitative differences, in contrast to our previous claim.\(^15\)

III

In this section we outline various methods of calculating proximity effects, with emphasis on the quasi-geometrical Green function technique, and then give results of a few simple calculations.

The theoretical problem is simple to state: one must solve the BCS-Gorkov (or Eliashberg) equations of superconductivity in the inhomogeneous situations shown in Fig. 1. Even in the familiar s-wave case this is a difficult task because the gap function will vary in space on the scale of the BCS coherence length $\xi$, and because one must know something about the normal state electronic wave functions of the system shown in Fig. 1. In practice, two approaches have been taken. One
is the MacMillan tunneling model,\textsuperscript{17} in which spatial variations of the gap are neglected (except that it may have different values in material A or material B). Free electron wave functions are assumed for materials A and B, and phenomenological tunneling matrix elements $T_{AB}$ are introduced to provide an amplitude for an electron to go from one material to the other. Because spatial variations of the gap are neglected this method is restricted to situations in which the thicknesses $d_A$, $d_B$ of the A and B materials satisfy $a < d_A < d_B$. In the MacMillan model, then, the gap in each material has the bulk value appropriate to the material and temperature plus a correction which is of order $T_{AB}$ when $T_{AB}$ is small. The MacMillan model has been applied to proximity effects involving non-s-wave superconductors,\textsuperscript{18} however this application did not properly include the pair-breaking effect of interfaces on anisotropic superconductivity.\textsuperscript{11}

An alternative approach to proximity effects, due to de Gennes,\textsuperscript{1,2} involves recasting the BCS equations as an integral equation for the gap $\Delta_0$, one finds for the s-wave case:

$$\Delta_0(x, x') = \int d^3p \, V(x, p) K_{s-wave}(x, x') \Delta_0(x', p)$$

(1)

Here $V(x)$ is the BCS pairing potential at position $x$. The $p$ integral is taken over the Fermi surface. To linear order in the gap function, the kernel $K$ may be expressed in terms of the normal-state Green functions of the problem. In the s-wave case (if time reversal invariance applies) de Gennes showed it may be expressed in terms of the density-density correlation function of the A-B system. The correct expression for $K$ (including the boundary condition at the A-B interface) may then be deduced from simple physical arguments and related to measurable normal-state quantities. This method may be applied to unconventional superconductivity.\textsuperscript{16,17} however, the kernel $K$ turns out to involve higher order correlation functions (current-current, density-current, density-spin current...) and the correct boundary condition is difficult to deduce, when spin-orbit coupling and a non-spherical Fermi surface are involved. In Refs. 16 and 19 the possibility that passage through even a non magnetic interface could alter the electronic spin as well as momentum was not correctly handled.

A method for computing proximity effects which avoids some of the difficulties of the previously described techniques has recently been devised.\textsuperscript{15} It is essentially a way to incorporate interfaces into the technique of quasiclassical Green functions devised by Eilenberger\textsuperscript{20} and used extensively to study $\text{He}_3$. In the quasiclassical Green function formalism one eliminates at the outset variations on the atomic length scale a $\text{km}$ and writes equations for quantities that vary on length scales $\sim \xi$. These equations contain all of the information needed to compute many physical quantities, including the spatial variation in the gap function, but are substantially less complex than the BCS equations. Because interfaces lead to spatial variations on length scales $\ll \xi$, the effect of an interface may be expressed as a boundary condition on the quasiclassical equations.\textsuperscript{12,22} The form of this boundary condition has been worked out, in the limit of a weakly transmitting boundary, for an interface which is planar, translationally invariant along the interface and otherwise arbitrary.\textsuperscript{15} To compute the boundary condition one must know the $S$ matrix appropriate to the interface. The $S$ matrix gives the amplitude for an incoming particle to be reflected or transmitted with or without its spin being flipped. The $S$ matrix would be difficult to compute, but may be parametrized in a simple way, and the constraints on $S$ imposed by time-reversal rotation or other symmetries may easily be determined.\textsuperscript{13}
is the MacMillan tunneling model, in which spatial variations of the gap are neglected (except that it may have different values in material A or material B), free electron wave functions are assumed for materials A and B, and phenomenological tunneling matrix element $T_{AB}$ is introduced to provide an amplitude for an electron to go from one material to the other. Because spatial variations of the gap are neglected this method is restricted to situations in which the thickness $d_A$, $d_B$ of the A and B materials satisfy $\langle \Delta_A \rangle < \langle \Delta_B \rangle$. In the MacMillan model, then, the gap in each material has the bulk value appropriate to the material and temperature plus a correction which is of order $T_{AB}$ when $T_{AB}$ is small. The MacMillan model has been applied to proximity effects involving non-s-wave superconductors; however this application did not properly include the pairbreaking effect of interfaces on anisotropic superconductivity.

An alternative approach to proximity effects, due to de Gennes, involves recasting the BCS equations as an integral equation for the gap $\Delta$; one finds for the s-wave case:

$$\Delta_{\text{corr}}(p) = \int d^3p' \left[ \text{V}(p,p') \right] K(p,p') \Delta_{\text{corr}}(p')$$

where $\text{V}(p,p')$ is the BCS pairing potential at position $x$. The $p'$ integral is taken over the Fermi surface. To linear order in the gap function, the kernel $K$ may be expressed in terms of the normal state Green functions of the problem. In the s-wave case (if time reversal invariance applies) de Gennes showed it may be expressed in terms of the density-density correlation function of the A-B system.

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The form of this boundary condition has been worked out, in the limit of a weakly transmitting boundary, for an interface which is planar, translationally invariant along the interface and otherwise arbitrary. To compute the boundary condition one must know the S matrix appropriate to the interface. The S matrix gives the amplitude for an incoming particle to be reflected or transmitted with or without its spin being flipped. The S matrix would be difficult to compute, but may be parametrized in a simple way, and the constraints on $S$ imposed by time-reversal, rotation or other symmetries may easily be determined.

Once the S matrix for an interface is given, one may solve the equations for the inhomogeneous system shown in Fig. 1, obtaining the form of the gap function, the superconducting $T_C$, etc. The simplest case is when the interface is invariant under time reversal and rotations about its normal. One may then parametrize the interface by a transmission matrix of the form:

$$S_{ij}(q) = c_{i} \cos \phi_{ij} + s_{ij} \cos \theta_{ij} + \epsilon_{ij} \sin \theta_{ij}$$

(2)

Here $\epsilon$ is the vector of Pauli matrices, $\epsilon_3$ is the unit normal to the interface, $\epsilon_3$ is a unit vector giving direction of the Fermi surface of material B, $\cos \phi = -\epsilon \cdot \epsilon_3$ and $\epsilon$ and $\epsilon_3$ are even functions of their argument. In the absence of spin-orbit coupling, $\epsilon_3 = 0$, in heavy fermion materials in which spin-orbit coupling is strong one expects $\phi = -\epsilon_3$.

Given an expression for the S-matrix, a general expression for the kernel in the inhomogeneous gap equation (to linear order in the gap functions) may readily be derived. The expression reduces to that given by de Gennes if one restricts to s-wave superconductivity. The full expressions are lengthy; the essential physics is contained in the expression for the case in which $\epsilon_3$ is in material B and $\epsilon$ is in material A:

$$K_{ij}(x,x') = [T_{ij}(x,x') + 2 \text{Im} \epsilon_3 \cos \phi_{ij} + \text{Im} \epsilon_3 \cos \theta_{ij}] + \epsilon_j \epsilon_3 \sin \theta_{ij}$$

(3)

Note that $\langle T_{ij} \rangle = [\epsilon_3^2 + \epsilon^2]$ is related the transmissivity of the interface that would be measured in e.g. a tunneling experiment, while $\langle \epsilon \rangle$ is difficult to determine by a normal state measurement. If there is spin orbit coupling, $\text{Im} \epsilon_3 \epsilon_3 \neq 0$. Evidently the kernel contains both terms which do not mix singlet and triplet superconductivity and terms which do. It is therefore possible for e.g. a triplet Cooper pair to cross an interface and convert itself to a singlet pair, or vice versa. The main conclusion of this work follows immediately: in the presence of strong spin-orbit coupling even an interface which is time reversal invariant and rotationally invariant about its normal will couple conventional s-wave superconductivity both to anisotropic singlet (e.g. d-wave) or to triplet superconductivity more or less as strongly as it couples s-wave to s-wave superconductivity.

Very similar arguments have been applied to the study of the Josephson effect between conventional and unconventional superconductors, with similar conclusions.\textsuperscript{12,13,14}

In any event, using the kernel given above one may calculate e.g. the form and magnitude of the order parameter induced in the surface layer (material A in Fig. 1) by bulk superconductivity in material B. We assume that material A is then (d << $\xi$) and is an s-wave superconductor with BCS pairing potential $\lambda_A$ and transition temperature $T_{BA}$ such that $T_{BA} > T_{CA}$. We linearize in the gap in material B and in the transmissivity of the interface. We consider three cases: (i) B is an s-wave superconductor with gap $\Delta_B$, (ii) B is a d-wave superconductor with a gap near the interface of the form $\Delta_B(q) = \Delta_B(q_0) \text{exp} \left[ -q_0 q_0 \right] \sin \theta(q_0) \sin \phi(q_0)$, where $\phi$ is a Pauli matrix and $\theta(q_0)$ is a spherical harmonic. (iii) B is a p-wave superconductor, in an axial\textsuperscript{3} state with $\epsilon$ vector normal to the interface, so $\Delta_B(q) = (q_0 + q_2 \gamma_2) \text{exp} \left[ -q_0 q_0 \right] \sin \theta(q_0) \sin \phi(q_0)$. The normal to the interface is chosen to be the axis about which spin and orbital angular momentum is quantized. In (ii) B is a given giving the preferred axis in spin space. The direction $\epsilon_3$ is determined by
the reflective properties of the interface, and will be discussed in more detail elsewhere.\textsuperscript{15} In (ii) and (iii) we have used only those components of the order parameter unaffected by the presence of a perfectly reflecting interface in the $xy$ plane. Other components would (to leading order in $T_J^a$) vanish within a distance $\sim \xi_0$ of the interface.

One then finds for $\Delta_A(y, x)$, the induced gap in material A, the following expressions:

$$\Delta_A(y, x) = \left( \frac{\lambda_A(\lambda_B)}{1 - \ln T_J^a} \right) C$$

\text{(4)}

Here $C$ is a constant. For case (i) $C = \int d^2 \mu [\Delta_A(\mu)]^2 \lambda_B$ for case (ii) $C = \int d^2 \mu \Delta_A(\mu) \lambda_B^2$, for case (iii) $C = 2 \int d \mu \Delta_A(\mu)$, where $f$ depends upon the angular average of $\bar{m}$ and $\bar{p}$. Note that the singlet-triplet coupling, in contrast to the singlet-antitangle coupling, involves the relative phase of the spin-independent and spin-dependent amplitudes. One can show that the symmetries of the problem (time reversal, unitarity, rotational invariance) do not require the singlet-triplet term to vanish; what determines the phase difference is not currently understood. However, assuming that any term not forbidden by symmetry will occur, one may conclude that except for a numerical factor, the form and magnitude of the induced gap in the layer is independent of the nature of the bulk superconductivity. The constant $C$ is largest in case (i), and may vanish in cases (ii) or (iii) if the bulk order parameter does not have components of the necessary symmetry. $C$ is smaller in case (ii) than in case (i). Thus, an anomalously weak proximity effect could be a signature of anisotropic superconductivity; however, $C$ is of the same order of magnitude in each of the cases (i)–(iii). Note also that the gap in material A is isotropic in momentum space, provided the pairing interaction in A is isotropic; thus $s$, $p$, or $d$-wave superconductivity in one material may be the presence of spin-orbit coupling, induce a $s$-wave gap in another material. Therefore the leakage of Cooper pairs into material A will occur and be measurable regardless of whether A is dirty (and hence pair-breaking for unconventional superconductivity) or not. From this it follows at once that the $T_J$ vs thickness curve (Fig. 2a) for an $s$-wave superconductor in proximity to an anisotropic superconductor will always be of the form shown in curve (2) in Fig. 2a, independent of the nature of the anisotropic superconductivity, although the numerical value of e.g. $\Delta_A$ will vary. Further results and details of the calculation will be given elsewhere.\textsuperscript{15}

In conclusion, proximity effect experiments do not seem to be a useful way of studying unconventional superconductivity, because no qualitative differences from conventional proximity effects are found. However, at present calculations have only been done for simple models; in particular, only specularly reflecting interfaces have been considered. Because diffuse scattering at an interface is to some extent pair-breaking for anisotropic superconductors, one expects in this case the magnitude of e.g. the leakage of Cooper pairs from an unconventional to a conventional superconductor to be reduced relative to that of $s$-wave to $s$-wave proximity effects; it is unclear whether this effect is large enough to warrant altering the pessimistic first sentence of the conclusion.
the reflective properties of the interface, and will be discussed in more detail elsewhere.\textsuperscript{13} In (ii) and (iii) we have used only those components of the order parameter unaffected by the presence of a perfectly reflecting interface in the xz plane. Other components would (to leading order in $\Delta_3a$) vanish within a distance $\sim \xi_0$ of the interface.

One then finds for $\Delta_3(x,y)$, the induced gap in material A, the following expression:

$$\Delta_3(x,y) = \frac{\Delta_3b}{1-\frac{\Delta_3b}{\Delta_3a}} C$$

Here $C$ is a constant. For case (i) $C = \int d^2\theta T_{1a}(\theta)T_{1b}$ for case (ii) $C = \int d^2\theta T_{1a}(\theta)T_{2b}$, and for case (iii) $C = \int d^2\theta T_{1a}(\theta)T_{2b}$, where $f$ depends upon the angular average of $\hat{n}$ and $\hat{z}$. Note that the singlet-triplet coupling, in contrast to the singlet-singlet coupling, involves the relative phase of the spin-independent and spin-dependent amplitudes. One can show that the symmetries of the problem (time reversal, unitarity, rotational invariance) do not require the singlet-triplet term to vanish; however, what determines the phase difference is not currently understood. However, assuming that any term not forbidden by symmetry will occur, one may conclude that except for a numerical factor, the form and magnitude of the induced gap in the layer is independent of the nature of the bulk superconductivity. The constant $C$ is largest in case (i), and may vanish in cases (ii) or (iii) if the bulk order parameter does not have components of the necessary symmetry. C is smaller in case (iii) than in case (ii). Thus, an anomalously weak proximity effect could be a signature of anisotropic superconductivity, however, $C$ is of the same order of magnitude in each of the cases (i), (ii), (iii). Note also that the gap in material A is isotropic in momentum space, provided the pairing interaction in A is isotropic; thus $\Sigma_1 \Sigma_2$, d-wave superconductivity in one material may, in the presence of spin-orbit coupling, induce a s-wave gap in another material. Therefore the leakage of Cooper pairs into material A will occur and be measurable regardless of whether $A$ is dirty (and hence pairbreaking for unconventional superconductivity) or not. From this it follows at once that the $T_c$ vs thickness curve (Fig. 2) for an s-wave superconductor in proximity to an anisotropic superconductor will always be of the form shown in curve (2) in Fig. 2, independent of the nature of the anisotropic superconductivity, although the numerical values of e.g. $\xi_0$ will vary. Further results and details of the calculation will be given elsewhere.\textsuperscript{15}

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