Parametric excitation of the $J = 2^+$ modes by zero sound in superfluid $^3$He-B

J.A. Saulsa and Ross H. McKenzieb

aDepartment of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, USA
bDepartment of Physics, Ohio State University, Columbus, OH 43210, USA

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We discuss order-parameter collective modes in weakly inhomogeneous states of superfluid $^3$He-B, i.e., states in which the scale of the inhomogeneities is considerably longer than the coherence length $\xi_0 = \eta_0^2 \pi T_c$, and the energy associated with the inhomogeneity is small compared to the condensation energy. The theory describes resonance phenomena between order-parameter modes and zero sound. We discuss two specific cases, both of which involve excitation of the $J = 2^+$ modes via a parametric field that lifts the selection rule due to particle-hole symmetry. In the case of a static superflow the modes with $J = 2^+, M = \pm 1$ couple to sound for $q \parallel H$, and should be observable as Zeeman states with a maximum absorption that scales as the square of the superflow velocity. The $J = 2^+$ modes may also be excited parametrically in a three-wave resonance process involving two zero-sound phonons. We summarize the nonlinear response theory for two-phonon excitation of these modes.

1. Introduction

In this paper we discuss collective modes in weakly inhomogeneous states of superfluid $^3$He-B, that is states in which the scale of the inhomogeneity is considerably longer than the superfluid coherence length $\xi_0 = \hbar \nu / 2 \pi T_c$. The theme of this paper is the excitation of collective modes that would otherwise be forbidden by a selection rule were it not for the coupling to sound via a parametric field. One example is the linear response of $^3$He-B with a superflow; a supercurrent breaks particle–hole symmetry, and thus provides a mechanism for coupling between zero sound and the $J = 2^+$ (real squashing) modes [1]. The other example we discuss is a three-wave resonance involving two phonons and the $J = 2^+$ mode, which is a nonequilibrium extension of the superflow coupling to nonlinear acoustics [2].

The order parameter for superfluid $^3$He is a tensor whose components, $d_{\alpha \beta}$, represent the nine possible amplitudes for pairs with total spin $S = 1$ and orbital angular momentum $L = 1$. The equilibrium B-phase is represented by the particular form,

$$d_{\alpha \beta} = \Delta(T) e^{i\phi} R_{\alpha \beta} [\hat{n}, \theta],$$

(1)

where $\Delta(T)$ is the real amplitude, $\phi$ is the global phase, and $R_{\alpha \beta}$ is an orthogonal matrix representing a rotation of the spin and orbital coordinates. The amplitude $\Delta(T)$ is fixed by the condensation energy. Excitations of the magnitude of the order parameter, or any deviation from the local equilibrium structure of $d_{\alpha \beta}$, are costly in energy. Thus, the amplitude is a stiff degree of freedom. The B-phase spontaneously breaks rotational symmetry in spin and orbital spaces, as well as gauge symmetry. The global phase, $\phi$, and the rotation matrix, $R_{\alpha \beta} [\hat{n}, \theta]$, represent soft degrees of freedom because the free energy of $^3$He is invariant under uniform gauge transformations and rotations—indeed under separate spin and space rotations were it not for the tiny dipolar energy. Thus, inhomogeneous states described by long wavelength variations of $\phi$ or $R_{\alpha \beta}$ cost little energy on the scale of the condensation energy, $U_c = \frac{1}{2} N(E_c) \Delta^2$. 
2. Symmetry and quantum numbers

If we neglect the nuclear dipole energy, then any rotation matrix in eq. (1) minimizes the free energy. This degeneracy is partially lifted by the dipole energy, which fixes \( \theta = 104^\circ \) [3]. However, the B-phase retains a high degree of residual symmetry. Note that if it had turned out that the dipolar energy favored the state with \( \theta = 0 \), then we would have \( d_{ia} \sim \delta_{ia} \), which is invariant under a simultaneous rotation of spin and orbital coordinates; the spectroscopic classification of the B-phase would be a pair state with \( S = 1 \), \( L = 1 \) and \( J = 0 \), where \( J = L + S \) is the total angular momentum. We would also identify the residual symmetry of the B-phase by the rotation group \( SO(3) \). The symmetry group and quantum numbers of the true B-phase remain intact even though \( \theta \neq 0 \). One simply recognizes that the correct generator is \( J = L + R^{-1} S \).

The order parameter describing an excitation of the condensate can be written,

\[
d_{ia} = \Delta R_{ia} + \delta d_{ia}(x, t).
\]

The collective modes of the order parameter described by \( \delta d_{ia} \) may be broadly classified into (i) Goldstone modes, excitations of the soft degrees of freedom, and (ii) ‘exciton’ modes, excitations associated with a deformation of the order parameter. Since the B-phase is isotropic (i.e., \( J = 0 \)) these excitations correspond to order-parameter fluctuations with quantum numbers \( J \) and \( M \), the projection of \( J \) along a fixed direction \( \hat{z} \). The spectrum of collective modes for the BW state has been investigated by many authors (see ref. [4] for original and complete references). There are two modes with the rotational symmetry of the BW state, i.e., \( \delta d_{ia} = d_{0} R_{ia} \). In the long wavelength limit they are,

\[
\delta d_{0}^+ = \text{Re} d_{0} \sim \frac{1}{\omega^2 - 4A^2},
\]

\[
\delta d_{0}^- = \text{Im} d_{0} \sim \frac{1}{\omega^2 - c^2 q^2}.
\]

The real mode, \( \delta d_{0}^+ \), is an amplitude excitation with an energy gap of \( 2\Delta \), while the imaginary mode, \( \delta d_{0}^- \), is an excitation of the phase of the order parameter, the Goldstone mode associated with the spontaneously broken gauge symmetry. The phase mode is identifiable with collisionless sound because of the relation between phase fluctuations and supercurrent, and the continuity equation connecting longitudinal current fluctuations with the density. Thus, the velocity in the dispersion relation of eq. (3) is nearly identical with the hydrodynamic sound velocity.

Modes with \( J = 1 \) also occur, but we do not discuss them here. The \( J = 2 \) modes also divide into ‘real’ and ‘imaginary’ modes,

\[
\delta d_{ia}^\pm = \sum_{M=-2}^{+2} D_M i_M^{(M)}(\hat{z}),
\]

with

\[
D_M \sim \frac{1}{\omega^2 - \omega(2^-, M)^2},
\]

and \( i_M^{(M)}(\hat{z}) \) is the eigenfunction for a mode with quantum numbers \( J = 2, M = (-2, \ldots, 2) \). In zero field (and \( q \to 0 \)) both sets of \( J = 2 \) modes are five-fold degenerate. This degeneracy is lifted by a magnetic field giving rise to a five-fold Zeeman splitting of each multiplet [5]. The spectrum of excitations is summarized in fig. 1. There is dispersion of the modes of the form, \( \omega_{\pm M}(q) = \omega_{\pm}^2 + c_{\pm M}^2 q^2 \), with velocities \( c_{\pm M} \sim \frac{\omega_{\pm}}{v_{\pm}} \); the dispersion partially lifts the degeneracy of each multiplet, even in zero field.

Ultrasonic spectroscopy of superfluid \(^3\)He-B is governed by two basic processes (in linear response): (i) pair breaking, in which a high-frequency phonon dissociates a Cooper pair, and (ii) resonant excitation of an excited pair state by the sound field. Below the pair-breaking threshold (\( \hbar \omega < 2\Delta(T) \)) thermal quasiparticles are relatively ineffective in damping sound because their density is very low. Resonant excitation of an order-parameter mode is then the dominant damping mechanism. However, the order-parameter modes are broadened by the
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Figure 1: The pair excitation spectrum for ^3He-B.

![Image of a graph showing the pair excitation spectrum for ^3He-B, with labels for different modes and Landau levels.](image)

Figure 2: Ultrasonic absorption spectrum for ^3He-B.

![Image of a graph showing the ultrasonic absorption spectrum for ^3He-B, with labels for different modes and resonances.](image)

thermally excited quasiparticles. Figure 2 is a sketch of the sound absorption spectrum of ^3He-B, showing resonant absorption into both J = 2 modes, as well as pairbreaking at frequencies above the gap edge. The widths of the absorption peaks are approximately the quasiparticle lifetime, 1/τ ≈ Δ.

The excitation spectrum in the vicinity of the gap edge may be complicated. Attractive f-wave pairing gives rise to bound states with J = 4 lying just below 2Δ. In addition, the J = 1^− modes, which have an excitation energy of 2Δ (at q = 0 and H = 0), may couple to sound in a magnetic field. As yet there is no definitive identification of either of these modes in ^3He-B. A detailed review of ultrasonic spectroscopy and the theoretical interpretations in terms of collective modes is contained in ref. [4].

An important feature of the absorption spectrum is that the coupling strength of the J = 2^+ mode to sound is several orders of magnitude smaller than that of the J = 2^− mode. This is a consequence of an approximate symmetry of the normal Fermi-liquid phase of ^3He, called particle–hole symmetry.

We discuss the particle–hole symmetry of the normal state by introducing a unitary operator, ℂ, with quantum numbers ξ = ±, that transforms a quasiparticle with excitation energy ξ > 0 and spin projection ↑ into a quasihole with energy ξ = −ξ and spin projection ↓, ℂaₜₜ′, ℂ⁺ = [iσₘ]_ₜₜ′aₜₜ′. Under the particle–hole transformation the low-energy effective Hamiltonian for the quasiparticle excitations remains approximately invariant, $\mathcal{H} \rightarrow \mathcal{H} + \text{small}$. An estimate of the relative size of the particle–hole asymmetry terms is given by

$$
\eta \approx \frac{N′(E_t)π T_c}{N(E_t)} \sim \left( \frac{T_c}{E_t} \right) \sim \text{few} \times 10^{-3}.
$$

Symmetry imposes selection rules for the allowed transitions. In the case of ^3He-B, rotational symmetry and particle–hole symmetry require,

$$
\Delta M = 0 \quad (\text{rotational symmetry}),
$$

$$
\Delta \xi \neq 0 \quad (\text{particle–hole symmetry}).
$$
for excitation of the collective modes via phonon absorption in zero field. Unless \( H \parallel q \) a magnetic field lifts the selection rule \( \Delta M = 0 \). Similarly, the small violation of particle–hole symmetry in the low-energy quasiparticle Hamiltonian removes the selection rule \( \Delta \mathcal{E} \neq 0 \), allowing the \( J = 2^+ \) modes to couple to sound, albeit weakly, with a coupling strength smaller by a factor of order \( \eta \sim (T_c/E_s) \) compared with that of the \( J = 2^- \) modes [6].

It is nevertheless instructive to consider how one might observe the \( J = 2^+ \) modes if particle–hole symmetry of the Hamiltonian were exact. To this end it is useful to formalize the particle–hole symmetry selection rules. Serene [7] has done so by considering the transformation properties of the density fluctuation operator, \( \delta n_{\text{op}} = \psi \psi - (\psi \psi) \), and the pair-field operator, \( \delta d_{\text{op}}^{(z)} = \mathcal{P}_{\alpha} (\psi \psi + \psi^\dagger \psi^\dagger) \), where \( \mathcal{P}_{\alpha} \) represents the \( S = 1, L = 1 \) projection of the pair field. The following operator relations can be obtained:

\[
(\mathcal{E} \delta n_{\text{op}}^{(z)})^{\dagger} = -\delta n_{\text{op}} \quad \text{and} \quad (\mathcal{E} \delta d_{\text{op}}^{(z)})^{\dagger} = \pm \delta d_{\text{op}}^{(z)}.
\]

The linear response of the condensate to an external field that excites the density can be formally written in terms of the Kubo function,

\[
X_{d_{\text{op}}}^{(z),\delta n}(\rho) = -\mathrm{Tr} \{ \rho [d_{\text{op}}^{(z)}, \delta n_{\text{op}}] \}_q \omega.
\]  

The operator relations imply the conditions,

\[
X_{d_{\text{op}}}^{(z),\delta n}(\rho) = \mp X_{d_{\text{op}}}^{(z),\delta n}(\rho^\dagger \rho^\dagger),
\]

so that the selection rule forbidding excitation of the \( J = 2^+ \) modes by sound follows if the density matrix, \( \rho \), is particle–hole symmetric. This is the case for the uniform B-phase (again, to the extent that the normal-state low-energy Hamiltonian is particle–hole symmetric).

The breaking of particle–hole symmetry can also be accomplished by preparing \(^3\text{He}-\text{B}\) in a state with superfluid flow. For uniform superflow the order parameter, \( d_{\text{in}} = \Delta R_{\text{in}} e^{\imath \mathcal{E} \mathbf{v}_s \cdot \mathbf{r}} \), is complex, and consequently the density matrix describing the flow state is no longer invariant under the operation \( \mathcal{E} \). In fact, the Kubo function satisfies,

\[
X_{d_{\text{in}}}^{(z),\delta n}(q, \omega; \mathbf{v}_s) = \mp X_{d_{\text{in}}}^{(z),\delta n}(q, \omega; -\mathbf{v}_s),
\]

and the selection rule on the \( J = 2^+ \) modes is lifted. Condition (10) implies that the coupling is linear in \( \mathbf{v}_s \). The coupling also depends on the direction of the flow relative to the propagation direction [1].

The propagation of sound is governed by a wave equation,

\[
(\omega^2 - c_s^2 q^2) \delta n = 2c_s^2 q^2 \mathcal{E} \Pi,
\]

where \( \mathcal{E} \Pi = (\hat{q}_x \hat{q}_y - \frac{1}{2} \delta_{\alpha\beta}) \Pi_{\alpha\beta} \) is the longitudinal component of the stress tensor, and \( c_s \) is the hydrodynamic sound velocity. This equation is applicable to nonlinear wave propagation as well as linear response. What is required is the constitutive equation relating the stress, \( \mathcal{E} \Pi \), to the density fluctuations and the internal degrees of freedom of the fluid. In order to obtain the constitutive equations we need a transport theory for the excitations and order parameter modes. The details of this theory are discussed elsewhere [8, 9]; what the theory provides is information about the quasiparticles through a distribution function, \( \delta g(\hat{\rho}; q, \omega) \), and about the condensate through the pair amplitude, \( \delta f(\hat{\rho}; q, \omega) \), both of which are functionals of the density and order-parameter fluctuations. Once obtained, the distribution function and pair amplitude allow us to calculate the constitutive equations via,

\[
\mathcal{E} \Pi(\delta n, \delta d_{\text{in}}) = \int \frac{d\Omega}{4\pi} P_{\alpha}(\hat{\rho} \cdot \hat{q}) \delta g(\hat{\rho}; q, \omega),
\]

where \( \delta f(\hat{\rho}; q, \omega) \), both of which are functionals of the density and order-parameter fluctuations. Once obtained, the distribution function and pair amplitude allow us to calculate the constitutive equations via the wave equation.

Consider \(^3\text{He}-\text{B}\) with a uniform texture and superflow in a magnetic field \( \vec{n} || H \parallel q \) (in this field orientation the Zeeman levels are not observable without a superflow). The constitutive equation determining the flow contribution to the stress is [1, 9]

\[
\left( \frac{\delta \Pi}{\delta n} \right) = \mathcal{A}(\omega, T) \left( \frac{v_t}{c_1} \right)^2 \left( \frac{p_t v_t}{\Delta} \right)^2 c_s^2 q^2 \times \{ \cos^2 \beta \mathcal{P}_{2,0}(\omega) + \alpha \sin^2 \beta \}
\]

\[
\times \left[ \mathcal{P}_{2,1}(\omega) + \mathcal{P}_{2,-1}(\omega) \right],
\]

\[
X_{d_{\text{in}}}^{(z),\delta n}(q, \omega; \mathbf{v}_s) = \mp X_{d_{\text{in}}}^{(z),\delta n}(q, \omega; -\mathbf{v}_s),
\]

where \( \mathcal{E} \Pi = (\hat{q}_x \hat{q}_y - \frac{1}{2} \delta_{\alpha\beta}) \Pi_{\alpha\beta} \) is the longitudinal component of the stress tensor, and \( c_s \) is the hydrodynamic sound velocity. This equation is applicable to nonlinear wave propagation as well as linear response. What is required is the constitutive equation relating the stress, \( \mathcal{E} \Pi \), to the density fluctuations and the internal degrees of freedom of the fluid. In order to obtain the constitutive equations we need a transport theory for the excitations and order parameter modes. The details of this theory are discussed elsewhere [8, 9]; what the theory provides is information about the quasiparticles through a distribution function, \( \delta g(\hat{\rho}; q, \omega) \), and about the condensate through the pair amplitude, \( \delta f(\hat{\rho}; q, \omega) \), both of which are functionals of the density and order-parameter fluctuations. Once obtained, the distribution function and pair amplitude allow us to calculate the constitutive equations via,
where
\[ P_{2,M}(\omega) = \left( \left( \omega + i/\tau \right)^2 - \omega_{2,M}^2 \right)^{-1}, \quad (14) \]

\[ \cos \beta = \hat{\theta}_s \cdot \hat{H}, \alpha \text{ is a constant, and } A(\omega, T) \text{ is a dimensionless function, nominally of order unity, except for } T \rightarrow 0 \text{ and } T \rightarrow T_c \text{ where it vanishes.} \]

Thus, the attenuation of sound, \( \alpha(\omega) = -q \operatorname{Im}(\delta P/\delta n) \), shows absorption features from the \( M = \pm 1 \) modes even though the quantization axis is along the direction of propagation. These features are characteristic of the coupling of sound to the \( J = 2^+ \) modes via the superflow field; they are absent from the intrinsic particle-hole asymmetry coupling. Thus, the flow coupling would also be identified by the magnitude of the absorption from the \( M = \pm 1 \) modes, which scales as \( v_s^2 \), and from the separation of the \( M = \pm 1 \) absorption peaks from the central \( M = 0 \) peak by the Zeeman energy.

The magnitude of the flow coupling is determined by the parameter,
\[ \eta_{\text{flow}} = A(p_i u_i / \Delta)(v_i / c_i), \quad (15) \]
and can be compared with that for the intrinsic particle-hole asymmetry. The maximum flow coupling is comparable to that of the central absorption peak for velocities of order
\[ v_s^* = \eta(c_i / v_i)(\Delta / p_i), \quad (16) \]
which is well below the bulk critical velocity. However, this result [9] is larger by a factor \( c_i / v_i \) from that of [1]. Recent attempts to detect the flow coupling of sound to the \( J = 2^+ \) modes in rotating \(^3\)He-B based on the ‘gyrosonic effect’ proposed in ref. [10] are inconclusive; possibly because the counterflow velocities obtained in the rotating cryostat are below \( v_s^* \).

3. Nonlinear acoustics

The dynamical analog of the static superflow coupling of sound to the \( J = 2^+ \) modes is the excitation of these modes via two sound waves, one of which is the parametric field, the other is the signal wave. Analysis of this problem requires a nonlinear theory of the coupling of collective modes. The energy densities needed in order to observe these nonlinear effects are typically small compared to the condensation energy density, \( U_c \). Such weak nonlinear effects should be compared with the energy density, probably of order \( U_c \), required for saturation or self-induced transparency of sound interacting resonantly with a \( J = 2^+ \) mode. An important signature of parametrically excited \( J = 2^+ \) modes via two sound waves is the shift in the resonance position away from the linear absorption feature even in zero field. One possible geometry for a three-wave resonance experiment is shown in fig. 3.

Two sets of transducers are shown, one pair generates and detects high-frequency sound waves (the signal wave) with \( \omega_i = \Delta \) and amplitude \( \delta n_1 \), while the other pair generates a lower frequency wave with \( 1/\tau \ll \omega_s < \Delta \) and amplitude \( \delta n_2 \). The intensity of the low-frequency wave (the pump wave) can be chosen to be relatively large,
\[ \frac{U_1}{\hbar \omega_1} < \frac{U_2}{\hbar \omega_2}, \quad (17) \]
in which case the pump wave can be treated as a

![Fig. 3. A possible geometry for a three-wave resonance in \(^3\)He-B.](image)
parametric field with a constant number of phonons.

The formulation of the nonlinear response theory is described elsewhere [8]. Here we summarize some of the results applicable to the problem of three-wave resonance. The central equations describing the parametric excitation of the $J = 2^+$ modes are the nonlinear constitutive equations for the stress,

$$
\delta \Pi(\omega) = \frac{1}{(1 + F_0^2) \Delta} \sum_M \int d\nu A_M(\omega; \nu; \omega - \nu) \times \delta n(\nu) D_M^+(\omega - \nu),
$$

and the equation of motion for the $J = 2^+$ modes,

$$
[(\omega + i/\tau)^2 - \omega_{2^+, M}^2]D_M^+ = \frac{6\Delta}{(1 + F_0^2) \lambda(\omega)} \times \int d\nu A_M(\nu; \omega; \nu; \omega)^* \times \delta n(\nu) \delta n(\omega - \nu),
$$

where $D_M^+(\omega)$ is the amplitude of the $J = 2^+, M$ mode, $\delta n(\omega)$ is the density fluctuation, and $A_M(\omega; \nu; \omega - \nu)$ is a coupling function that can be calculated from the quasiclassical transport theory. In the three-wave resonance experiment the density is represented by

$$
\delta n = N_1 e^{i(q_1 x - \omega_1 t)} + N_2 e^{i(q_2 x - \omega_2 t)},
$$

where $N_1$ and $N_2$ are slowly varying envelopes for the signal and pump wave, respectively. The signal wave response has the form,

$$
\frac{\delta \Pi(\omega_1)}{N_1} = |N_2|^2 \chi^{(3)}(\omega_1, \omega_2; \omega_1 + \omega_2)
+ \chi^{(3)}(\omega_1, \omega_2; \omega_1 - \omega_2),
$$

where,

$$
\chi^{(3)}(\omega, \nu; \omega - \nu) = \frac{6/5}{(1 + F_0^2)^3} \times \sum_M \frac{|A_M(\omega, \nu; \omega - \nu)|^2}{A(\omega - \nu)[(\omega - \nu + i/\tau)^2 - \omega_{2^+, M}^2]}.
$$

This response function describes the two resonant absorption processes shown diagrammatically in fig. 4. The first process is excitation of the $J = 2^+$ mode by two-phonon absorption, which gives a sharp peak in the signal-wave absorption spectrum at a temperature determined by the resonance conditions, $\omega_1 + \omega_2 = \omega_{2^+}$, $q_1 + q_2 = Q_{2^+}$, and the dispersion relations for the modes. Note that the magnitude of the absorption peak depends on the intensity of the pump wave. The calculated two-phonon absorption peak is indicated in fig. 4, well separated from the central, linear absorption peak. Also, shown in fig. 4 is a sharp absorption peak corresponding to the conversion of a high-frequency phonon into a low-frequency phonon and a $J = 2^+$ excitation. This process is stimulated Raman scattering, and it occurs when the resonance conditions, $\omega_1 - \omega_2 = \omega_{2^+}$, $q_1 - q_2 = Q_{2^+}$, are satisfied. Note that if the signal wave is the low-frequency wave, then stimulated Raman scattering would amplify the signal wave as high-frequency pump wave phonons convert to low-frequency phonons. The optimum conditions for detecting a three-wave resonance involving the $J = 2^+$ modes are indicated by the maximum signal-wave absorption,

$$
\frac{\alpha(\omega_1)}{q_1} = \frac{\Delta c}{c_i} \approx \frac{|A(\omega_1, T)|^2}{(1 + F_0^2)^2} \left(\frac{\Delta \lambda}{\lambda}\right) \left(\frac{U_2}{U_1}\right).
$$

![Fig. 4. Signal wave absorption spectrum for $\omega_1/2\pi = 35.8$ MHz, $\omega_2/2\pi = 3.26$ MHz, and $U_2/U_1 = 0.1$.](image)
The dependence on pressure is largely determined by the fermi-liquid interaction, $F_0$, which varies between 10 and 100 from low to high pressures. Thus, the two-phonon absorption is expected to be largest at low pressures. Similarly, the intrinsic width of the resonance is due to thermally excited quasiparticles; thus, the magnitude of the absorption peak is larger (but narrower) at lower temperatures. The nonlinear absorption peak scales linearly with the energy density of the pump wave. The limiting factor on the power level of the pump wave is heating. Also the condition $U_2 \ll U_c$ is required for the validity of our perturbation theory. It should be mentioned that the coupling function $A_M(\omega, T)$ depends smoothly on temperature and frequency, and that the coupling is also a function of the propagation directions of the two sound waves. A detailed analysis can be found in ref. [8].

Recently, Torizuka et al. [11] detected the two-phonon excitation of the $J = 2^+$ mode at low pressures. These authors report an absorption peak appearing at the appropriate three-wave resonance condition as the pump wave intensity is increased, and that the peak absorption scales linearly with the pump wave energy density. These are the principle signatures of the two-phonon absorption process.

Finally, we want to point out that these three-wave processes in which one wave is treated as a parametric field are perhaps the simplest nonlinear acoustic excitations that can be studied. There are many aspects of nonlinear acoustics in superfluid $^3$He that are unexplored, either theoretically or experimentally.

References