Models for Superfluid ^{3}He in Aerogel

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Evidence of superfluidity of ^{3}He in 98%-porous aerogel has been found recently in two experiments. A microscopic model of the aerogel as a weakly inhomogeneous anisotropic scattering medium is shown to be in semi-quantitative agreement with experiments.

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The effect of impurities on a conventional superconductor is a well studied problem [1]. The corresponding effect for unconventional superfluid systems is experimentally much less studied because it is difficult to arrange bulk impurities into liquid ^{3}He, which is the only superconductor/superfluid with a well identified unconventional pairing state. This situation recently changed because of two experiments that find evidence of superfluidity of ^{3}He in very porous aerogel. A torsional oscillator experiment [2] and NMR experiment [3] show that the transition temperature and the amplitude of the superfluid state are suppressed relative to bulk ^{3}He. We discuss possible models that can be used to understand these results. We study in particular a homogeneous scattering model, where the basic assumption is that the aerogel acts as a homogeneous scatterer of the ^{3}He quasiparticles. We find that a strictly homogeneous and isotropic scattering model can explain only a part of the observations, but allowing for corrections due to inhomogeneity and anisotropy, the model seems qualitatively consistent with the experiments.

In both experiments the aerogel fills only 2% of the total volume ($V = 0.02$), and its surface to volume ratio is $A = 260,000\,\mathrm{cm}^{-1}$ [4]. Using these numbers alone we can crudely estimate that the material consists of a network of one-dimensional strands having diameter $4V/A = 3$ nm. The distance between strands is $\sqrt{4\pi V}/A = 20$ nm, and the mean free path is $\ell = 4/A = 150$ nm. Alternatively, if the structure consists of two-dimensional walls, their distance is $d = 2/A = 80$ nm. In order to analyze the superfluidity of ^{3}He in the aerogel, these lengths need to be compared with the coherence length $\xi_0 = \hbar v_F/2\pi k_B T_{c0}$. Here T_{c0} is the transition temperature in bulk ^{3}He, v_F the Fermi velocity; ξ_0 is a function of pressure varying between 16 nm (melting pressure) and 77 nm (zero pressure).

Because the volume fraction of the aerogel strands (including an inert layer of ^{3}He atoms on the strands) is small, we neglect all effects that are linear in the volume fraction. In particular, we assume that the density, the Landau Fermi-liquid parameters, and the coupling constant of the pairing interaction are unchanged from the bulk. The changes of these parameters are of the same order of magnitude as the volume fraction because they arise from processes of relatively high energy and short length scale [5]. Much larger effects on superfluidity arise from processes in the immediate vicinity of the Fermi surface. Scattering of quasiparticles from the aerogel strands modifies the superfluid state within the distance ξ_0, and causes an effect that is proportional to the ratio ξ_0/ℓ. In conventional s-wave superconductors the effect of nonmagnetic scattering is to shorten the coherence length. But, non-magnetic scattering is also pair-breaking in p-wave superfluids like ^{3}He, and leads to a suppression of T_c and the magnitude of the condensate order parameter.

A realistic theoretical description of superfluidity in aerogel is complicated because the strand spacing has the same order of magnitude as the superfluid coherence length. In order to develop a tractable theory, we consider two limiting cases. One is the homogeneous scattering model (HSM), where the scattering is assumed to be completely delocalized instead of being concentrated on the strands. The other extreme is that the scattering is strongly localized to some regions, which represent a collective effect of several strands. As the simplest model in this limit, we consider ^{3}He in a slab with diffusely scattering impermeable walls.

In the HSM we additionally assume that the scattering medium is isotropic, *i.e.* ℓ is independent of the direction of quasiparticle momentum. We also neglect magnetic scattering because it does not seem to be important for the effects we consider. A convenient property of the isotropic HSM is that both the Ginzburg-Landau (GL) theory and Leggett's theory of NMR [6] have the same form as in pure ^{3}He. Only the parameters of these theories have different values. Thus, the results of the calculations can be expressed using these parameters.

The Ginzburg-Landau theory is formulated in terms of a free energy functional of the 3×3 matrix order parameter, $A_{\mu i}$, where μ represents the spin components and i represents the orbital components of the pair state. The 'bulk' terms are [7,8]

$$\begin{aligned} f_{\rm bulk} = {} & \alpha A^*_{\mu i}A_{\mu i} + \beta_1|A_{\mu i}A_{\mu i}|^2 + \beta_2(A_{\mu i}A^*_{\mu i})^2 \\ & +\beta_3 A^*_{\mu i}A^*_{\nu i}A_{\nu j}A_{\mu j} + \beta_4 A^*_{\mu i}A_{\nu i}A^*_{\nu j}A_{\mu j} \\ & +\beta_5 A^*_{\mu i}A_{\nu i}A_{\nu j}A^*_{\mu j}\ . \end{aligned} \tag{1}$$

The material coefficients (α, β_i, etc.) are calculated using the quasiclassical theory [5] in the weak-coupling approximation. Without further approximations one finds [9]

$$\alpha = \frac{N(0)}{3}[\ln\frac{T}{T_{c0}} + 2\sum_{m=0}^{\infty}(\frac{1}{2m+1} - \frac{1}{2m+1+x})], \quad (2)$$

where $x = v_F/2\pi T\ell_{\rm tr}$, $\ell_{\rm tr}$ is the transport mean free path and $N(0)$ the density of states at the Fermi surface.

The superfluid transition is determined by the condition $\alpha(T_c) = 0$. Scattering suppresses T_c so that the relative transition temperature T_c/T_{c0} is a function of $\ell_{\rm tr}/\xi_0$. This suppression of T_c is compared with experiments in Fig. 1 assuming that $\ell_{\rm tr}$ is a pressure-independent constant. The major part of the suppression is accounted for by the HSM, but there also is a systematic deviation with pressure. An alternative way to apply the HSM is to use a pressure dependent $\ell_{\rm tr}(p)$ that exactly reproduces the measured $T_c(p)$. We will use this approach in further calculations of the HSM although the required p dependence of $\ell_{\rm tr}$ is larger than we estimate from on the p dependence of the Fermi momentum $\hbar k_F$.

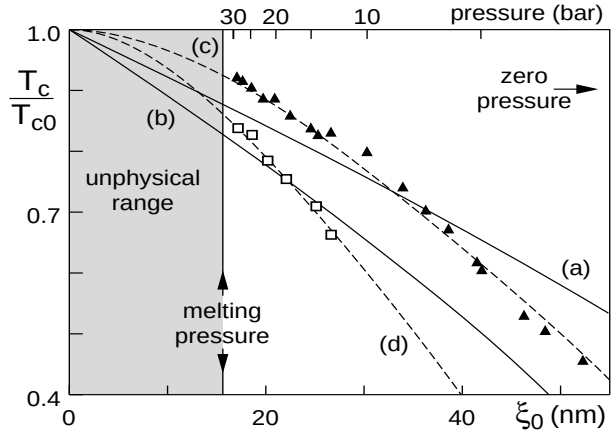

Fig. 1 The transition temperature in aerogel relative to that in bulk, T_c/T_{c0}. The horizontal axis is the coherence length $\xi_0 = \hbar v_F/2\pi k_B T_{c0}$, which is a monotonic function of pressure. The experimental results are from [2] (triangles) and [3] (boxes). The solid lines correspond to the homogeneous scattering model (HSM) at mean free paths $\ell_{\rm tr} = 320$ nm (a) and 230 nm (b), and the dashed lines correspond to slabs of thicknesses $d = 105$ nm (c) and 74 nm (d).

The same data is also analyzed using the slab model [10], which gives a surprisingly good fit. We can understand the difference in the pressure dependence of the two models as follows. The slab geometry causes only small suppression [$\propto (\xi_0/d)^2$] at small ξ_0/d because the superfluid state in the middle of the slab is not much affected by the scattering at the walls. In the HSM the scattering centers are distributed homogeneously; there are no preferential sites so that the transition is more strongly suppressed ($\propto \xi_0/\ell_{\rm tr}$). The broad distribution of pore sizes in aerogel [11] suggests that there are regions where superfluidity is weakly suppressed at small ξ_0. In this respect the slab model looks more realistic than the HSM. However, the "slabs" are not isolated. The regions of large and small pore sizes are strongly coupled by the high transition probability for quasiparticles moving through the relatively open aerogel structure. This coupling is necessary in order to understand the rather sharp transition seen experimentally. Thus, ^{3}He in aerogel has characteristics of both models: the superfluid transition is sharp (as in HSM), but is suppressed in proportion to $(\xi_0/d)^2$, characteristic of a more inhomogeneous scattering geometry.

For the fourth order coefficients, β_i, we make the additional assumption that only s-wave scattering is important, and obtain

$$\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\end{pmatrix} = a\begin{pmatrix}-1/2\\ 1\\ 1\\ 1\\ -1\end{pmatrix} + b\begin{pmatrix}0\\ 1\\ 0\\ 1\\ -1\end{pmatrix} \quad (3)$$

$$a = \frac{N(0)}{15(\pi T)^2}\sum_{m=0}^{\infty}(2m+1+x)^{-3}$$

$$b = \frac{N(0)v_F}{36(\pi T)^3\ell}\left(\frac{k_F^2\sigma}{2\pi} - 1\right)\sum_{m=0}^{\infty}(2m+1+x)^{-4}.$$

Note that in b the cross section σ of an individual scattering center and the density of such centers n_s do not combine to form a function of the mean free path $\ell = (\sigma n_s)^{-1}$ alone, as they do in all other formulas. The undetermined quantity $k_F^2\sigma/2\pi - 1$ is bounded by -1 (Born limit) and $+1$ (unitary limit) in the s-wave scattering approximation. It turns out that the GL results for a given ℓ are not very sensitive to σ.

The free energies of the superfluid phases are expressed in terms of α and β_i. For example, the polar, planar, and B phases have $f = -k\alpha^2/(4k\beta_{12} + 4\beta_{345})$, with $k = 1$, 2, and 3, respectively, and the A phase has $f = -\alpha^2/4\beta_{245}$, where $\beta_{ij...} = \beta_i + \beta_j + ...$ We find that for arbitrary ℓ and σ the weak-coupling, HSM with isotropic scattering predicts the B phase to be stable at low pressures where strong-coupling corrections are small. This differs from the slab model where an arbitrarily small strong coupling correction can stabilize the A phase at all pressures. Thus, strongly anisotropic ℓ, as is present in the slab model, could stabilize the A phase. However, we will show below that such a strong anisotropy is in contradiction with experiments.

There is another route to stabilization of the A-phase at low pressures. In the limit $k_F^{-1} \ll L_a \lesssim \xi_0$, where L_a is the correlation of the strands, the aerogel medium is nearly isotropic on the scale of the pairing correlations, but the scattering by a strand may be strongly anisotropic. In this limit we recover the GL functional of the isotropic HSM. However, the relative stability of the

p-wave states, which is determined by the fourth-order GL parameters, is sensitive to the anisotropy of the scattering from individual strands even when all orientations of the strands are contained within the size of a pair. Anisotropic backscattering, preferentially perpendicular to the strands, can stabilize the A-phase low pressures, where the B-phase is otherwise stable [12].

Experiments give evidence for the A phase, or at least an equal-spin pairing state, at pressures above 13 bar [3]. This is because no change is seen in the magnetic susceptibility in entering the superfluid phase [Eqs. (10) and (11) below]. Other measured quantities do not seem to discriminate between A and B type phases.

Other terms of interest in the GL theory are the coefficients of the gradient energy, K, with anisotropy parameter, γ, the magnetic field energy, g_z, and the dipole-dipole energy, g_d [7]. With the same assumptions used to calculate the β_i's, the HSM gives,

$$K = \frac{N(0)v_F^2}{30(\pi T)^2}\sum_{m=0}^{\infty}(2m+1+x)^{-3} \tag{4}$$

$$\gamma = 3 + \frac{5v_F}{6\pi T\ell}\frac{\sum_{m=0}^{\infty}(2m+1)^{-1}(2m+1+x)^{-3}}{\sum_{m=0}^{\infty}(2m+1+x)^{-3}} \tag{5}$$

$$g_z = \frac{\tilde{\gamma}^2 N(0)}{6(\pi T)^2(1+F_0^a)^2}\sum_{m=0}^{\infty}(2m+1+x)^{-3} \tag{6}$$

$$g_d = \frac{2\pi}{5}(\tilde{\gamma}N(0))^2R^2\left[\sum_{m=0}^{\epsilon_c/2\pi T}(2m+1+x)^{-1}\right]^2, \tag{7}$$

where $\tilde{\gamma}$ is the gyromagnetic ratio, F_0^a is the Fermi-liquid exchange interaction parameter, R^2 is a renormalization constant for the dipole energy, and ϵ_c a high-energy cutoff [6]. The dipole-dipole coupling constant g_d is unchanged by scattering because the explicit dependence on ℓ and the implicit dependence of ℓ through the reduction of T_c cancel.

The measurable quantities that are derivable from the GL coefficients are the superfluid density ρ_s, the change in magnetic susceptibility $\delta\chi = \chi - \chi_N$, and the frequency shift $\delta\omega$ in transverse NMR. For homogeneous A- and B-phases these quantities are [6,8]

$$\rho_{sA} = 8m_3^2K\Delta_A^2[\gamma+1-(\gamma-1)(\hat{\mathbf{l}}\cdot\hat{\mathbf{v}})^2] \tag{8}$$

$$\rho_{sB} = 8m_3^2(\gamma+2)K\Delta_B^2 \tag{9}$$

$$\delta\chi_A = 0 \qquad (\hat{\mathbf{d}}\perp\mathbf{H}) \tag{10}$$

$$\delta\chi_B = -2g_Z\Delta_B^2 \tag{11}$$

$$\delta\omega_A = 2\tilde{\gamma}g_D\Delta_A^2[(\hat{\mathbf{l}}\cdot\hat{\mathbf{d}})^2-(\hat{\mathbf{l}}\cdot\hat{\mathbf{H}})^2]/\chi_N H \quad (\hat{\mathbf{d}}\perp\mathbf{H}) \tag{12}$$

$$\delta\omega_B = 15\tilde{\gamma}g_D\Delta_B^2|\hat{\mathbf{n}}\times\hat{\mathbf{H}}|^2/2\chi_N H \qquad (\theta = 104^\circ), \tag{13}$$

where $\hat{\mathbf{H}} = \mathbf{H}/H$ and $\hat{\mathbf{v}}$ are the the directions of the magnetic field and superfluid velocity, $\Delta_A^2 = \alpha/4\beta_{245}$, $\Delta_B^2 = \alpha/(6\beta_{12}+2\beta_{345})$, m_3 is the ^{3}He atomic mass, and χ_N is the susceptibility in the normal phase (including the inert surface layer of ^{3}He). In the A phase $\hat{\mathbf{d}}$ and $\hat{\mathbf{l}}$ are the spin and orbital anisotropy axes. The B phase spin-orbit rotation matrix $R_{\mu i}(\hat{\mathbf{n}},\theta)$ is parametrized by an axis $\hat{\mathbf{n}}$ and an angle θ.

The values of ρ_s for the models are compared with experiment in Fig. 2. If the flow is large enough to orient $\hat{\mathbf{l}}$, one expects $\hat{\mathbf{l}} \parallel \hat{\mathbf{v}}$. Otherwise one should take some average over $\hat{\mathbf{l}}$ in (8). Both alternatives, as well as the B phase (9) and the slab model, give ρ_s of the right order of magnitude, but all fail to reproduce the fast drop with decreasing pressure [2]. The models give $\rho_s \sim (T_c - T)$ for $T \lesssim T_c$ but experimentally $\rho_s \propto (T_c - T)^n$ with $n \approx 1.5$.

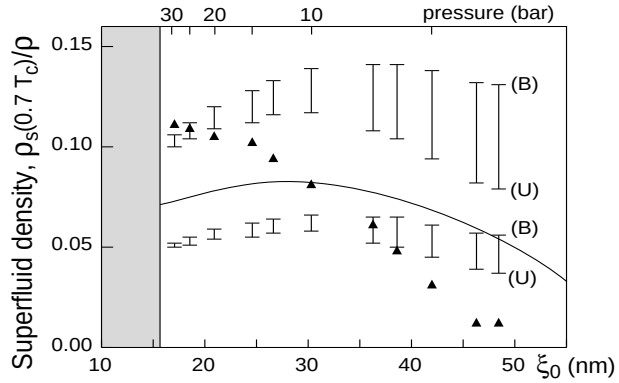

Fig. 2 The superfluid density $\rho_s(T = 0.7T_c)/\rho$ as a function of ξ_0 or pressure. The triangles denote the experimental result [2]. The upper bars are the results of HSM for the B phase or the A phase with $\hat{\mathbf{l}} \perp \mathbf{v}$. The lower bars are for A phase with $\hat{\mathbf{l}} \parallel \mathbf{v}$. The curve is the result of the slab model (d = 105 nm) for flow parallel to the plane of the slab. The theoretical results are linear extrapolations from T_c to $0.7T_c$. The ends of the bars correspond to Born (B) and unitary (U) scattering limits.

These differences in ρ_s can be qualitatively understood by inhomogeneities induced by the aerogel. Let us consider a local superfluid density that varies around an average value ρ_{s0}. This leads to a reduction of the measurable macroscopic ρ_s. If the local variation $\delta\rho_{s0}$ is small, then $\delta\rho_s \propto -(\delta\rho_{s0})^2/\rho_{s0}$. Thus, the reduction is relatively largest at small ρ_{s0}, *i.e.* near T_c and at low pressure, where the difference between the models and the experiment mostly exists. Conversely, we can conclude that ρ_s has to be relatively homogeneous at high pressures when $T \lesssim 0.7T_c$ because the experimental and model ρ_s have the same magnitude. This is evidence against strict interpretation of the slab model, where the superfluid density perpendicular to the slab vanishes, and would lead to additional reduction of the measurable ρ_s for randomly oriented slabs.

In order to understand the NMR frequency shift, let us first recall the case in bulk liquid. The A-phase $\hat{\mathbf{l}}$ and the B-phase $R_{\mu i}(\hat{\mathbf{n}},\theta)$ are determined only by the dipole-dipole energy. These "Leggett configurations" have $\hat{\mathbf{l}} \parallel \hat{\mathbf{d}} \perp \mathbf{H}$ and $\theta = 104^\circ$, $\hat{\mathbf{n}} \parallel \mathbf{H}$. As a consequence,

the maximum positive frequency shift is observed in bulk A phase (12), while there is no shift in the bulk B phase (13). The experimental frequency shifts in aerogel are compared with the models in Fig. 3. The maximum frequency shift in the A phase is plotted for both the HSM (12) (bars) and the slab model (line). We argue below that the difference between the models and the experiment is due to anisotropy of the aerogel.

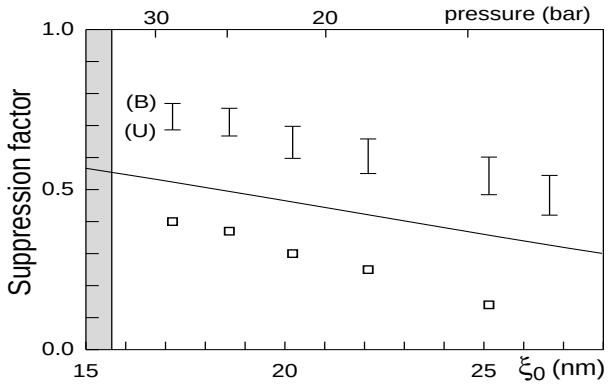

Fig. 3 The suppression factor for the shift of the NMR resonance frequency, defined as $\chi_N \delta\omega_A$ at fixed $t = T/T_c \lesssim 1$ divided by the same quantity in pure ^{3}He at $T = tT_{c0}$. The boxes denote the experimental result [3]. The maximum A-phase shift for HSM is shown by bars, and for the slab model by the line ($d = 74$ nm, field in the plane of the slab). The ends of the bars correspond to Born (B) and unitary (U) scattering limits.

If the scattering rate is dependent on the direction of the quasiparticle momentum, the α-term in the GL functional (1) is modified to $\alpha_{ik} A^*_{\mu i} A_{\mu k}$. The matrix α_{ik} has the same form as above (2) except that $\ell_{\rm tr}^{-1}$ is replaced by a matrix $(\ell_{\rm tr}^{-1})_{ik}$. The direction of such anisotropy varies from one location to another. Thus we can say that the aerogel constitutes a random field. This field directly couples to the *orbital* part of the order parameter, *i.e.* to the $\hat{\mathbf{l}}$ vector in the case of the A phase. The randomness implies that the orbital direction is correlated only over a finite length $L_{\rm o}$ [13]. The *spin* part ($\hat{\mathbf{d}}$ vector in the A phase) feels the random field only via the dipole-dipole interaction. Thus the correlation length of the spin part, L_s, cannot be smaller than the dipole length $\xi_d = \sqrt{K/g_d}$. Because of the weakness of the dipole-dipole interaction, ξ_d is large, ≈ 10 μm.

Suppose that the orbital correlation length $L_{\rm o}$ were small compared to L_s. Then the dipole-dipole energy would be essentially averaged out leaving only a fluctuation $\propto (L_{\rm o}/L_s)^{3/2}$ relative to the value in homogeneous superfluid. The frequency shift, which is directly proportional to the dipole-dipole energy, would be reduced correspondingly. Because experimentally there is no large reduction, we can conclude that $L_{\rm o}$ cannot be much smaller than $\xi_d \sim 10$ μm.

Let us check if our estimate of $L_{\rm o}$ is in agreement with the microscopic structure of aerogel. We model an aerogel strand as a cylinder that scatters quasiparticles diffusively. This gives $(\ell^{-1})_\perp - (\ell^{-1})_\parallel \approx \frac{3}{8}(\ell^{-1})_{\rm average}$, and allows an estimate the magnitude $\delta\alpha$ of the random field (2). Its consequences depend on the parameter $\lambda = \delta\alpha L_a^2/K$, where L_a is the correlation length of the random field. If $\lambda \gg 1$, the orbital part will everywhere be oriented by the local anisotropy ($L_{\rm o} \sim L_a$). In the opposite extreme $\lambda \ll 1$, the anisotropy is largely averaged out and the orbital part is coherent over distance $L_{\rm o} \sim L_a/\lambda^2$ [13]. We assume that the strand directions are correlated over the same length scale as their spacing; $L_a \approx 20$ nm. This represents a weak inhomogeneity, $\lambda \approx 0.1$. The expression for $L_{\rm o}$ above is rather uncertain because it depends on the third power of the poorly known L_a, but it is feasible that $L_{\rm o}$ might be almost as large as 10 μm, in agreement with the estimate above.

In conclusion, the HSM, with corrections from inhomogeneity and anisotropy, provides a basic understanding of many properties of superfluid ^{3}He in aerogel, e.g. the suppressed but sharp T_c, the stability of new superfluid phases at low pressures, the superfluid density, and the NMR frequency at small tipping angles.

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