Sound Propagation and Transport Properties of Liquid $^{3}$He in Aerogel

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Abstract
Superfluid $^{3}$He confined in aerogel offers a unique chance to study the effects of a short mean free path on the properties of a well defined superfluid Fermi liquid with anisotropic pairing. Transport coefficients and collective excitations, e.g. longitudinal sound, are expected to react sensitively to a short mean free path and to offer the possibility for testing recently developed models for quasiparticle scattering at aerogel strands. Sound experiments, together with a theoretical analysis based on Fermi liquid theory for systems with short mean free paths, should give valuable insights into the interaction between superfluid $^{3}$He and aerogel.

A model for liquid $^{3}$He in aerogel based on a random distribution of short-ranged potentials acting on $^{3}$He quasiparticles has been shown to account semi-quantitatively for the reduction of the transition temperature, $T_{c}$, and the suppression of the superfluid density, $\rho_{s}(T)$. Although the order parameter for $^{3}$He in aerogel is not firmly identified, measurements of the magnetization indicate that the low pressure phase is an ESP state. A transverse NMR shift is observed and is roughly consistent with that of an axial state even though the B-phase is stable in pure $^{3}$He at these pressures. The addition of a small concentration of $^{4}$He, which coats the aerogel strands, induces a suppression of the magnetization for $T < T_{c}^{\text{aerogel}}$, indicative of a non-ESP state like the BW phase. Thus, the stability of the superfluid state of $^{3}$He in aerogel is quite sensitive to the detailed interaction between $^{3}$He and the aerogel strands. Calculations of the free energy for $^{3}$He in aerogel which include anisotropic and magnetic scattering, and the effects of orientational correlations of the aerogel strands, confirm the sensitivity of the superfluid phases to the interaction between $^{3}$He and the aerogel.$^{1,3}$

The high porosity of the aerogel implies that the silica structure does not significantly modify the bulk properties of normal $^{3}$He. The dominant effect of the aerogel structure is to scatter $^{3}$He quasiparticles moving at the
bulk Fermi velocity. If the coherence length, $\xi_0 = \hbar v_f / 2\pi T_c$, is sufficiently long compared to the average distance between scattering centers then a reasonable starting point is to treat the aerogel as a homogenous scattering medium (HSM) described by a mean-free path $\ell$. In this article we discuss the effects of a short mean free path on some of the transport properties of liquid $^3$He. The calculations presented below for sound propagation assume the BW phase is stable; however, the propagation and damping of low-frequency sound are expected to be qualitatively similar for other phases.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{$p$-$T$ phase diagram vs. $\ell$. The geometric mean-free path is estimated to be 1750 Å. The data is from Ref. 4.}
\end{figure}

The superfluid transition temperature for $^3$He in aerogel is suppressed by quasiparticle scattering off the aerogel structure. In the HSM model the suppression of $T_c$ is given by the Abrikosov-Gorkov formula,

$$\ln\left(\frac{T_0}{T_c}\right) = \Psi\left(\frac{1}{2} + \frac{1}{2} \frac{\xi_0 T_c}{\ell}\right) - \Psi\left(\frac{1}{2}\right),$$

with the pair-breaking parameter given by $\xi_0/\ell$. Figure 1 shows the calculated aerogel transition temperature for several values of the mean-free path. The pressure dependence of the calculated phase diagram is determined by the pressure dependence of the bulk transition temperature and the Fermi velocity via $\xi_0 = \hbar v_f / 2\pi T_c$. The experimental data for $T_c(p)$ is from Matsumoto, et al.\cite{4} for an aerogel with $\approx 98\%$ open volume. For this porosity the typical diameter of the aerogel strands is $d \simeq 30$ Å and the mean distance between strands is $D \simeq 325$ Å, which should be compared with the bulk coherence length $\xi_0 \simeq 700$ Å at $p = 1$ bar. The geometric mean-free path of the
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...determined from the surface to volume ratio, $S \approx 2.6 \times 10^5$ cm$^{-1}$, is $\ell_{\text{geom}} = \frac{3r^2}{k} S \approx 1750$ Å. The calculations show a mean-free-path of this order gives good agreement with the phase diagram at low temperatures and low pressures, i.e., for $\xi(p) = \hbar v_f/2\pi k_B T_c(p, \ell) > D \approx 325$ Å, which corresponds to pressures $p \lesssim 15$ bar. Also note that the mean-field phase diagram shows a zero-temperature phase transition as a function of pressure determined by $\xi_0(p_c) = 0.28 \ell$.

The transport properties of $^3$He should also be strongly affected by scattering from the aerogel. In the normal state the quasiparticle distribution function, $\phi(p, R; \epsilon, t)$, satisfies the Boltzmann-Landau transport equation,

$$\frac{\partial \phi}{\partial t} + v_f \cdot \nabla \phi + \left( \frac{\partial \phi_0}{\partial \epsilon} \right) \frac{\partial E}{\partial t} = I[\phi],$$

where $E$ is the effective potential acting on the quasiparticles and $I[\phi]$ is the collision integral. The effective potential consists of the external driving potentials and the internal potentials resulting from quasiparticle interactions. In pure liquid $^3$He the quasiparticle lifetime is determined by inelastic collisions between quasiparticles and is of order $\tau_{\text{in}} \approx 1\mu s$ at $p = 15$ bar. However, in aerogel the mean time between scatterings by the aerogel is $\tau_{\text{cl}} = \ell/v_f \approx 4$ ns at the same pressure. Thus, for temperatures below $T_\star \approx 16$ mK the collision rate is dominated by quasiparticle scattering by the aerogel: $I_{\text{el}} = -\frac{1}{\tau_{\text{cl}}} (\phi - \langle \phi \rangle_{\psi})$. This leads to strong reduction in the thermal conductivity and viscosity, and to an increase in the damping of hydrodynamic sound. The static transport coefficients exhibit cross-over behavior dictated by the scattering rate; the thermal conductivity and shear viscosity scale as

$$\kappa = \frac{1}{3} C_v v_f^2 \tau \sim \begin{cases} \frac{1}{T} & T \gtrsim T_\star, \\ \frac{1}{T} \frac{N_f v_f^2}{\epsilon^2} \tau & T < T_\star, \end{cases} \quad \eta = \frac{1}{15} N_f v_f^2 \frac{2}{\epsilon^2} \tau \sim \begin{cases} \frac{1}{T^2} \text{const} & T > T_\star, \\ \frac{1}{T^2} & T < T_\star. \end{cases}$$

above and below $T_\star$. Recent speculations that the low-temperature phase of $^3$He in aerogel for $p < p_{c\tau}$ is not described by Fermi-liquid theory$^4,5$ can be tested by measuring these transport coefficients.

At low frequencies a compressible aerogel will move nearly in phase with the $^3$He density and longitudinal current mode; sound propagates but it is damped by the viscous coupling of the $^3$He to the aerogel, $\alpha_1 = \frac{\rho v_f^2}{\rho c_1} \eta$, where $c_1$ is the hydrodynamical sound velocity and $\rho$ is the mass density of $^3$He. The viscous damping of hydrodynamic sound saturates for $T < T^\ast$ at $\alpha_1/q \approx \frac{2}{3} \omega \tau_{\text{cl}}/(1 + F_0^2)$. At higher frequencies the impedance mismatch between $^3$He and the aerogel sound mode leads to an increasingly out-of-phase motion of $^3$He excitations and aerogel. Hydrodynamic sound may become
overdamped and reemerge as a diffusive mode. The frequency at which the cross-over from damped hydrodynamic sound to an overdamped diffusive mode occurs depends on the elastic compliance of the aerogel and the microscopic details of the coupling between $^3$He and the aerogel strands. Here we assume the frequency is above this cross-over, in which case the hydrodynamic mode is an overdamped diffusive mode, $\omega = -iD_s q^2$, where $D_s = c_1^2 \tau$ is the acoustic diffusion constant with $1/\tau = (1 + F_3^q/3)/\tau_{cl}$. At still higher frequencies, $\omega \tau \gg 1$, zero sound can propagate, albeit with relatively high attenuation, $\alpha_0 \approx 1/c_0 \tau \approx 10^4 \text{ cm}^{-1}$ at $p = 10 \text{ bar}$. The regime for propagating zero sound is also pushed to higher frequencies, $\omega/2\pi > v_f/2\pi \ell \approx 42 \text{ MHz}$ at $p = 10 \text{ bar}$.

![Graph showing suppression of $\rho_s(T)$ by scattering at $p = 10 \text{ bar}$ with $F_3^q = 8.6$. Results in Born approximation are shown for bulk $^3$He-B and mean-free paths $\ell = 1600$ to $3000 \text{ Å}$. The inset shows the 4$^{th}$ sound velocity for the same values of the mean-free path.](image)

Fig. 2 - Suppression of $\rho_s(T)$ by scattering at $p = 10 \text{ bar}$ with $F_3^q = 8.6$. Results in Born approximation are shown for bulk $^3$He-B and mean-free paths $\ell = 1600$ to $3000 \text{ Å}$. The inset shows the 4$^{th}$ sound velocity for the same values of the mean-free path.

The collective mode spectrum of superfluid $^3$He in aerogel is also expected to show significant changes compared to bulk $^3$He.$^6$-$^8$ A typical example is low frequency sound in superfluid $^3$He ($\omega \ll \Delta/\hbar$). In bulk $^3$He one has collisionless zero sound and hydrodynamic first sound with nearly the same velocities, $c_0^2 \approx c_1^2 = \frac{1}{3}\left(1 + F_3^q\right)(1 + \frac{1}{3}F_3^q)v_f^2$, and small damping by quasiparticle-quasiparticle scattering (for reviews on sound and collective modes in $^3$He see e.g. Refs. 9-11). In aerogel, on the other hand, one expects a behavior which is similar to sound in $^3$He confined to small channels.$^{12}$ Sound will be weakly damped for $q\ell \gg 1$, ceases to be a well defined mode for $q\ell \approx 1$, and reappears for $q\ell \ll 1$ as fourth sound with a temperature
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dependent velocity, $c_4 \simeq \sqrt{\rho_s(T)/\rho c_1}$. Fig. 2 shows the effects of elastic scattering on the superfluid density. Note the reduction in $\rho_s/\rho$ at $T = 0$. For a mean-free path of $\ell = 1800 \text{Å}$ less than 50% of the $^3$He mass density contributes to $\rho_s(T = 0)$. The inset shows the fourth sound velocity neglecting damping.

\[ \text{Fig. 3 - Spectral function of the density response } \chi(\mathbf{q}, \omega) \text{ for } q\ell = 0.001 \ldots 10. \text{ The results are for } T/T_c = 0.8, 1/\tau_c T_c = 0.01, F_0^s = 50 \text{ and } F_1^s = 8.6; \text{ the corresponding zero sound mode is at } \omega/v_f q = c_0/v_f = 8.1. \]

Measurements of sound propagation in $^3$He-aerogel should provide a sensitive test of the HSM model. We study the spectrum of longitudinal sound in this model by calculating the linear response of the $^3$He density, $\rho(\mathbf{q}; \omega)$, to a driving force of wavevector $\mathbf{q}$ and frequency $\omega$. Our driving force will be a scalar field described by a potential, $\delta U_{\text{ext}}(\mathbf{q}; \omega)$, which couples to the $^3$He density. More realistic “experimental driving forces” require more elaborate calculations, which are not called for given the present experimental status. The calculation follows the quasiclassical version of the method of Betbeder-Matibet and Nozières. In the low frequency limit one has to solve Boltzmann-Landau transport equations for the branches of particle-like and hole-like excitations with distribution functions $\delta \phi_{B_1,B_2}(\mathbf{p}, \mathbf{R}; e, t)$. We keep the dominant Landau parameters, $F_0^s$ and $F_1^s$, and obtain an effective scalar potential, $\delta \tilde{u}(\mathbf{q}; \omega)$, and longitudinal vector potential, $\delta \tilde{u}(\mathbf{q}; \omega) = \mathbf{v}_f \cdot \delta \mathbf{A}(\mathbf{q}; \omega)$. The collision terms in the HMS model have the form, $I_{B_1,B_2} = -\frac{1}{\tau_0} \left( \delta \phi_{B_1,B_2} - \left< \delta \phi_{B_1,B_2} \right>_{\text{FS}} \right)$, where
\[ \tau(\epsilon) = \ell/v(\epsilon), \text{ and } v(\epsilon) = v_f \sqrt{\epsilon^2 - |\Delta|^2/\epsilon} \] is the energy dependent quasiparticle velocity in the superfluid state. In the low frequency limit the transport equations have to be supplemented by Landau’s self-consistency equations for the effective potentials, \( \delta \tilde{u} \) and \( \delta \tilde{a} \), and the particle conservation law, \( \dot{\rho} + \nabla \cdot \mathbf{j} = 0 \). In this limit we can ignore the less important self-consistency equation for the amplitude, \( \delta |\Delta| \), of the order parameter. The five coupled linear equations for the distribution functions, the effective potentials, and the phase, \( \delta \Psi(q;\omega) \), of the order parameter can be solved, and will be described elsewhere.

![Graph](image.png)

Fig. 4 - Temperature dependence of the imaginary and real (inset) parts of the spectral function \( \chi(q,\omega) \) for \( T/T_c = 0.25 \ldots 0.85 \). The results are shown for \( q\ell = 0.1, \ qv_f = 0.001 T_c, \ F_0^s = 50 \) and \( F_1^s = 8.6 \).

The calculated crossover from weakly damped zero sound to weakly damped fourth sound is shown in Fig. 3. We display the frequency dependent spectral function, \( -\Im \chi(q,\omega) \), for various wavelengths at fixed temperature and elastic scattering time. One can see clearly the transition from zero sound at \( v_f q \gg 1/\tau_{el} \) to fourth sound at \( v_f q \ll 1/\tau_{el} \). Fig. 4 shows the real and imaginary parts of the response function, \( \chi(q,\omega) \), at various temperatures and fixed \( \tau_{el} \). Because of the short mean free path, \( q\ell = 0.1 \), the zero sound resonance with wavevector \( q \) is overdamped in the normal state and just below \( T_c \). The damping decreases exponentially in the superfluid state for \( T \ll \Delta \) because of the freezing out of thermally excited quasiparticles; and, as can be seen from Fig. 4, a well defined zero sound mode develops.
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